

系所組別： 工程科學系甲乙丙丁戊己組

考試科目： 工程數學

考試日期： 0223，節次： 3

※ 考生請注意：本試題不可使用計算機

1. Solve

$$\frac{d}{dt} \begin{Bmatrix} y \\ z \end{Bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} y \\ z \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix} e^{-2t}, \text{ with } \begin{Bmatrix} y \\ z \end{Bmatrix} (0) = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix} \quad (25\%)$$

2. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + xe^{-t}$, $0 \leq x \leq 1$, with $u(0, t) = 1$, $u(1, t) = 2$,

$$u(x, 0) = 0, \quad \frac{\partial}{\partial t} u(x, 0) = x \quad (25\%)$$

3. A Solid is enclosed by the surfaces $0 \leq x, y \leq a$, $x^2 + y^2 \leq a^2$ and $z = x^2 + y^2$.

Calculate the total surface area of the solid. (25%)

4. Let α and β be distinct positive number. Calculate

$$\int_0^{2\pi} \frac{1}{\alpha^2 \cos^2 \theta + \beta^2 \sin^2 \theta} d\theta, \quad \beta/\alpha < 1 \quad (25\%)$$