系所組別：工程科學系乙組

考試科目：計算機數學 考試日期：0223，節次：3

※ 考生請注意：本試題不可使用計算機

1. Let R be a relation on a set X. Define

$R^{-1} = \{(y, x) \mid (x, y) \in R\}$,

$\rho(R) = R \cup \{(x, x) \mid x \in X\}$

$\sigma(R) = R \cup R^{-1}$

(1) (5%)For the relation $R_1 = \{(1,1), (1,2), (3,4), (4,2)\}$. Find $\rho(R_1)$ and $\sigma(R_1)$.

(2) (5%)Show that $\rho(R)$ is reflexive.

(3) (5%)Show that $\sigma(R)$ is symmetric.

(4) (5%)How can we quickly determine whether a relation R is a function by examining the matrix of R?

2. (1) (10%) Use the formulas

$s_1 = 2$, $s_n = s_{n-1} + 2n$ for all $n \geq 2$, to write an algorithm that computes s_n

(2) (10%)Give a proof that your algorithm is correct.

(3) (10%)Solve the recurrence relation defined by s_n.

3. (20%) Represent the postfix expression A B + C D * E F / - - A * as (1)a binary tree and (2)write the prefix form, (3)the usual infix form and (4)the fully parenthesized infix form of the expression, and (5)find the value of the postfix expression if A=1, B=2, C=3, D=4, E=6, F=3.

(背面仍有題目,請繼續作答)

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4. Refer to the following adjacency matrix of a weighted graph. Suppose that the vertices represent offices. An edge connects two offices if there is a communication link between the two. Notice that any office can communicate with any other either directly through a communication link or by having others relay the message.

	A	B	C	D	E	F	G	H	I
A	0	337	1846	1464	2704	∞	∞	∞	∞
B	337	0	∞	1235	∞	∞	∞	∞	2342
C	1846	∞	0	802	867	849	740	621	∞
D	1464	1235	802	0	∞	∞	1391	∞	1121
E	2704	∞	867	∞	0	∞	187	∞	1258
F	∞	∞	849	∞	∞	0	144	∞	∞
G	∞	∞	740	1391	187	144	0	184	1090
H	∞	∞	621	∞	∞	∞	184	0	946
I	∞	2342	∞	1121	1258	∞	1090	946	0

(1) (10%)Use Djijkstra's shortest path algorithm to find the shortest path from vertex H to vertex A.

(2) (10%)Find a minimum spanning tree for the graph. You can either use Prim's algorithm or Kruskal's algorithm. But you must specify which algorithm you use.

(3) (5%)What is the maximum number of communication links that can be broken with communication among all offices still possible?

(4)(5%)Show a configuration in which the maximum number of communication links are broken with communication among all offices still possible.