※ 考生請注意：本試題不可使用計算機

1. 20%) Let $\boldsymbol{x}$ be an eigenvector of both $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices. Is $\boldsymbol{x}$ also an eigenvector of $(A-\alpha B)$, where $\alpha \in R$? Explain.

2. 20%)

(a) Find the general solution to the following equation: $\dfrac{d^2 y}{dt^2} - 4\dfrac{dy}{dt} + 4y = 8e^{2t}$.

(b) Find the solution to the above equation for $y(0) = 3$ and $\left.\dfrac{dy}{dt}\right|_{t=0} = 4$.

3. 20%)

(a) (10%) Let Σ be a piecewise smooth closed surface bound a region M. Show that volume of $M = \dfrac{1}{3}\iint_{\Sigma} \vec{R}\cdot\vec{n}dA$, where $\vec{R} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{n}$ is the surface normal of Σ.

(b) (10%) Find the work done by the force $\vec{F} = 8xy^3 z\vec{i} + 12x^2y^2z\vec{j} + 4x^2y^3\vec{k}$ acting along the helix $\vec{r}(t) = 2\cos(t)\vec{i} + 2\sin(t)\vec{j} + t\vec{k}$ from (2,0,0) to (0,2,π/2).

4. 20%) Use the Fourier series method to solve the problem:

$$u_t = 4u_{xx} \qquad 0 < x < 2, t > 0$$

$$u(0,t) = u(2,t) = 0, \qquad t > 0$$

$$u(x,0) = 2[1-\cos(4\pi x)], \qquad 0 < x < 2$$

5. 20%) Consider the function of complex variable

$$f(z) = \frac{e^{az}}{(e^z+1)}, \qquad 0 < a < 1$$

(a) Locate the *singularities* and evaluate the *residues* of $f(z)$.

(b).Evaluate the following integral using the *Residue Theorem*

$$I = \int_{-\infty}^{\infty} \frac{e^{ax}}{e^x+1}dx, \qquad 0 < a < 1.$$