

※ 考生請注意：本試題不可使用計算機

1. Please find the derivate of the following functions. (20%)

(1) $y = 2\cot^2(\pi x + 2)$ (2) $y = 5^{\sqrt{x}}$ (3) $y = \ln\left(\sqrt{\frac{e^x+1}{e^x-1}}\right), x > 0$
 (4) $y = \log_5 \frac{x\sqrt{x-1}}{2}$

2. Please determine the limit for the following functions. (15%)

(1) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{2x^2}$ (2) $\lim_{x \rightarrow 0} \frac{a^x - a^{\tan x}}{x^3}$ (3) $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x})$

3. As shown in the Figure 1, rainbows are formed when light beam strikes raindrops and is reflected (at point B) and refracted (at points A and C). The law of refraction states that $(\sin \alpha / \sin \beta) = k$. The angle of deflection is given by $D = \pi + 2\alpha - 4\beta$. Let $D' = \frac{dD}{d\alpha}$, $0 \leq \alpha \leq \frac{\pi}{2}$. Please find the minimum angle of deflection (represents as k). (15%)

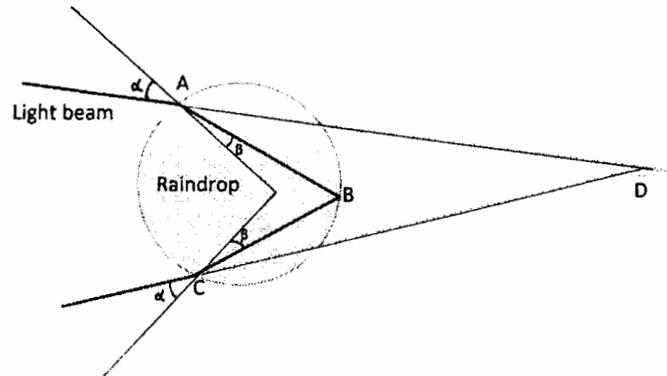


Figure 1

4. Please evaluate the integral of the following functions. (25%)

(1) $\int (x+1)\sqrt{2-x} dx$ (2) $\int \frac{1}{\sin^3 x} dx$ (3) $\int \frac{x^2+1}{x(x+1)^3} dx$
 (4) $\int xe^x \sin x dx$ (5) $\int_0^{\pi/2} \int_0^3 re^{-r^2} dr d\theta$

(背面仍有題目,請繼續作答)

系所組別：環境工程學系丙組

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5. Water is flowing through a channel which has a cross-section in the shape of isosceles trapezoid, as shown in Figure 2. If the two legs and bottom have the same length 1. Please find the maximum cross-sectional area. (15%)

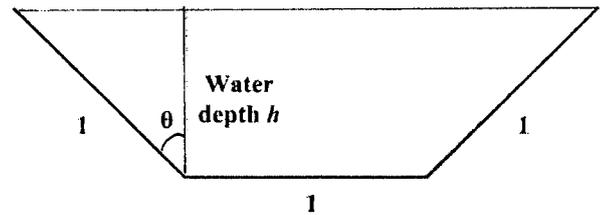


Figure 2

6. Please use a line integral to find the area of an ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (10%)