系所組別：環境工程學系甲、乙組

考試科目：工程數學　　　　　　　　　　　　　　　　　　　考試日期：0223，節次：3

※ 考生請注意：本試題不可使用計算機

1. Please solve the given differential equations. (35%)

    (1) $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 5y = \sin(x)$ 　　　　(2) $(x+2)^2\dfrac{d^2y}{dx^2} - (3x+6)\dfrac{dy}{dx} + 4y = 3x+2$

    (3) $\dfrac{d^2y}{dx^2} - (\dfrac{x}{x-1})\dfrac{dy}{dx} + \dfrac{1}{x-1}y = 0$ 　　(4) $y'' + e^{3y}(y')^3 = 0$, y is a function of x.

    (5) $\dfrac{d^2x}{dt^2} + 4x = -5\sin 2t + 3\cos 2t$, for $x(0) = -1$, $x'(0) = 1$

2. Please find the eigenvalues and eigenfunctions for the following boundary-value problems. (15%)

    (1) $y'' + \lambda y = 0$, $y'(0) = 0$, $y(L) = 0$ 　　　　(2) $y'' + \lambda y = 0$, $y(-\pi) = 0$, $y(\pi) = 0$

    (3) $y^{(4)} - \lambda y = 0$, $y'(0) = 0$, $y''(\pi) = 0$, $y'''(0) = 0$, $y(\pi) = 0$

3. Please solve the partial differential equation $\dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2} - hu$ with the given conditions. (20%)

    IC: $u(x,0) = 0$, $0 < x < \pi$ 　　　　　　BC: $u(0,t) = 0$, $u(\pi,t) = u_0$

4. Consider the initial value problem given below, please list the computational steps necessary to explicitly approximate $y(x=a)$ and $y'(x=a)$ using the fourth-order Runge-Kutta method with step size $h=a$. (15%)

$$y'' + 3x(y')^2 + y = 2x, \quad y(0) = 1, \quad y'(0) = 2$$

5. Consider a second-order boundary-value problem given below. Suppose the interior mesh points y_i, y_{i+1} and y_{i+2} of the interval $[x=1, x=2]$ are approximated at $x_i=1.25$, $x_{i+1}=1.5$ and $x_{i+2}=1.75$ using the finite difference method in matrix form as $\{y\} = [A]^{-1}\{B\}$. Please determine the matrix $[A]$ and vector $\{B\}$. (15%)

$$x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + 3y = 0, \quad y(1) = 5, \quad y(2) = 0$$