

系所組別： 生物醫學工程學系甲、乙、丁組

考試科目： 工程數學

考試日期： 0225，節次： 3

1. (25%) For a matrix $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$

a. Find a nonsingular matrix Q , let $\hat{A} = Q^{-1}AQ$ be a diagonal matrix. (10 points)

b. Find the inverse matrix A^{-1} (5 points)

c. $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

If $x_1^2 + x_2^2 = 1$, find the maximal and minimal values of $y_1^2 + y_2^2$ (10 points)

2. (20%) Find the solution of following differential equation

$$\ddot{y}(t) + \omega_0^2 y(t) = \cos \omega_1 t, \quad y(0) = 0, \dot{y}(0) = 0$$

a. When $\omega_1 \neq \omega_0$ (10 points)

b. When $\omega_1 = \omega_0$ (10 points)

3. (10%) Find the mean and variance of the following data set.

$$[1, 2, 3, 4, 5, 6]$$

4. (15%) Find the Fourier coefficient of the periodic function $f(x)$

$$f(x) = \begin{cases} -1 & \text{if } -\pi < x < 0 \\ 1 & \text{if } 0 < x < \pi \end{cases} \text{ and } f(x+2\pi) = f(x) \quad (15 \text{ points})$$

5. (10%) A vector field $\mathbf{v} = 3xz\mathbf{i} - 2xy\mathbf{j} - yz^2\mathbf{k}$

a. Find divergence of the vector field. (5 points)

b. Find the curl of the vector field. (5 points)

6. (10%) Find the improper integral $\int_0^{\infty} \frac{dx}{1+x^4}$ (10 points)

7. (10%) Find the center of gravity of a mass of density $f(x, y) = 1$

The triangle with vertices $(0, 0)$, $(b, 0)$, (b, h) (10 points)