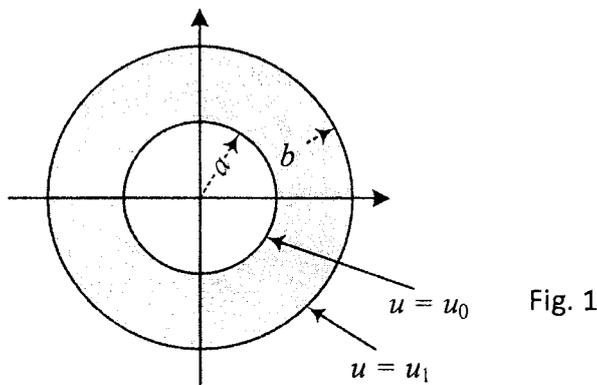


※ 考生請注意：本試題不可使用計算機

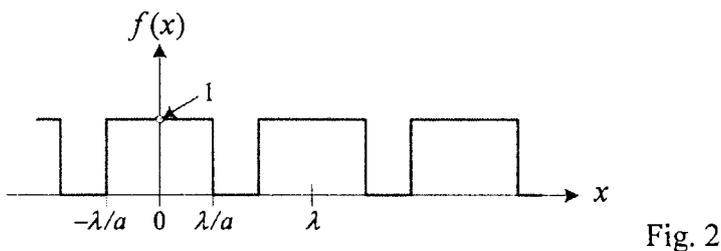
1. (12%) The temperature $u(r)$ in the circular ring shown in Fig. 1 is determined from the boundary-value problem

$$r \frac{d^2 u}{dr^2} + \frac{du}{dr} = 0, \quad u(a) = u_0, \quad u(b) = u_1,$$

where u_0 and u_1 are constants. Please solve for $u(r)$.



2. (12%) Compute the Fourier series components for the periodic function shown in Fig. 2 and express this function as a Fourier series.



3. (14%) A string is stretched and secured on the x -axis at $x = 0$ and $x = \pi$ for $t > 0$. If the transverse vibrations take place in a medium that imparts a resistance proportional to the instantaneous velocity, then the wave equation takes on the form

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + 2\beta \frac{\partial u}{\partial t}, \quad 0 < \beta < 1, \quad t > 0.$$

Find the displacement $u(x, t)$ if the string starts from rest and from the initial displacement $f(x)$.

4. (12%) The differential equation $m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = 0$ can be used to describe a damped simple

harmonic motion. Its solution can be written as the form of $x(t) = x_m e^{-\alpha t} \cos(\omega t + \phi)$, where x_m is the amplitude of the damped oscillator. Please solve this differential equation and find the α and ω in terms of m, b, k .

(背面仍有題目, 請繼續作答)

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5. (5 %) (1) $\vec{F} = [\sin^2 x, -y \sin 2x, 5z]$, S the surface of the box $|x| \leq a, |y| \leq b, |z| \leq c$. Please evaluate the integral $\iint_S \vec{F} \cdot \vec{n} \, dA$.

(5 %) (2) $\vec{F} = [y, z^2, x^3]$, C the intersection of $x^2 + y^2 = 1$ and $z = y + 1$. Please evaluate the integral $\oint_C \vec{F} \cdot \vec{r}' \, ds$. (The line integral is clockwise as seen by a person standing at the origin.)

(5 %) (3) Please find a parametric representation of the following curve:

Circle $\frac{1}{2}x^2 + y^2 = 1, z = y$.

6. (5 %) (a) Please give the definitions of Hermitian, Skew-Hermitian, and Unitary Matrices.

(5 %) (b) Please prove that the eigenvalues of a skew-Hermitian matrix are pure imaginary or zero.

7. (5 %) (a) Please find all the singular points and the corresponding residues for $\frac{\sin z}{z^6}$.

(5 %) (b) Please find the Cauchy principal value for $\int_{-\infty}^{\infty} \frac{x+5}{x^3-x} \, dx$

(5 %) (c) Please integrate $\frac{\cos z}{z^n}$ for $n = 1, 2, \dots$ counterclockwise around C: $|z| = 1$.

8. (5 %) (a) Find and sketch or graph the image of the given region under the given mapping.

Region: $\ln 3 < x < \ln 5$, mapping: $w = e^z$ ($z = x + iy$).

(5 %) (b) Find all points at which the mapping $z^2 + az + b$ are not conformal ($z = x + iy$).