1. Solve the following initial value problem

$x^2 y'' - xy' - 24y = 0$, $y(1) = 15$, $y'(1) = 0$. (10%)

2. Find the eigenvalues and eigenfunctions of

$y'' + \lambda y = 0$, $y(0) = y(1)$, $y'(0) = y'(1)$. (10%)

3. Solve the following system of ODEs by Laplace transform

$y_1'' = 4y_2 - 4e^t$, $y_2'' = 3y_1 + y_2$, $y_1(0) = 1$, $y_1'(0) = 2$, $y_2(0) = 2$,

$y_2'(0) = 3$. (10%)

4. Find the directional derivative of $f = x^2 + y^2 - z$ at P: $(1, 1, -2)$ in the direction of $\mathbf{a} = [1, 1, 2]$. (10%)

5. Using Green's thoerem, evaluate $\int_C \mathbf{F(r)} \cdot d\mathbf{r}$ counterclockwise around the boundary curve C of the region R, where

$\mathbf{F} = \left[e^{x+y}, e^{x-y} \right]$, R the triangle with vertices $(0, 0)$, $(1, 1)$, $(1, 2)$. (10%)

6. Find the Fourier cosine integral representation of the even function

$f(x) = \begin{cases} x & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases}$. (10%)

7. Solve for z of $\ln z = 2 + \dfrac{1}{4} \pi i$ in the complex plane, where $i = \sqrt{-1}$. (15%)

8. Evaluate the integral $\oint_C \dfrac{z+2}{z-2} dz$, C: $|z - 1| = 2$ counterclockwise, $z = x + iy$. (10%)

9. Evaluate the integral $\displaystyle\int_{-\infty}^{\infty} \dfrac{dx}{(x^2 + 4)^2}$. (15%)