系所組別： 電腦與通信工程研究所丙組
考試科目： 電磁數學

※ 考生請注意：本試題不可使用計算機

1. (15%) Find a suitable integration factor $\sigma(x)$ or $\sigma(y)$, and use it to find the general solution of the differential equation

$$dx + (3x - e^{-2y})dy = 0$$

2. (15%) Solve the following differential equation

$$y'' - (x^2+1)y = 0 \quad \text{with} \quad y(0) = 0 \quad \text{and} \quad y'(0) = 1$$

3. (20%) Consider a particle of mass m, carrying an electrical charge q, and moving in a uniform magnetic field of strength B. The field is in the positive z direction. The equations of motion of the particle are

$$mx'' = qBy'$$
$$my'' = -qBx'$$
$$mz'' = 0$$

where $x(t)$, $y(t)$, $z(t)$ are x, y, z displacements as a function of the time t. Find the general solution for $x(t)$, $y(t)$, $z(t)$.

4. (25%) Mark each of the following statements True (T) or False (F).

(a) If A and B are two $n \times n$ non-invertible matrices, then AB is also non-invertible.

(b) If a square matrix A is not invertible, then $A + I$ is invertible, where I is the identity matrix of the same size as A.

(c) Let W be a subspace of an inner product space V, and $W^{\perp}$ be the orthogonal complement of W. In general, we have $W \cup W^{\perp} = V$.

(d) We can transform any linear independent set of non-zero vectors into an orthogonal set of vectors by the Gram-Schmidt process.

(e) Let T be a linear transformation from a vector space V to a vector space W. Define a transformation $S : \mathbf{v} \rightarrow T(\mathbf{v}) + \mathbf{w}_o$ from V to W, where $\mathbf{w}_o$ is a constant vector in W. Then S is also a linear transformation from V to W.

5. Suppose that A is a 6×4 real matrix of rank 4. Let $W = A^T A$ and $S = AA^T$.

(a) (10%) Find the ranks of W and S, respectively.

(b) (5%) Explain why $\lambda = 0$ is an eigenvalue of S.

(c) (10%) What is the (algebraic) multiplicity of the eigenvalue $\lambda = 0$ of S?