※ 考生請注意：本試題不可使用計算機

1. (20%) Solve the following initial value problems.

(a) $y' = \frac{2y}{x} + x^2 e^x$, $y(2) = 0$

(b) $y' + 3x^2 y = x e^{-x^3}$, $y(0) = -1$

2. (13%) Solve the following initial value problem.

$y'' + 3.7y' = 0$, $y(-2) = 4$, $y'(-2) = 0$

3. (17%) As we know, the Fourier series of $f(x)\ \forall x \in [L, -L]$ can be expressed as

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right).$$

Now please answer the following questions.

(a) (3%) In what condition of $f(x)$, we should use Fourier integral to express $f(x)$ instead of Fourier series?

(b) (7%) Under the condition you express in (a), derive the Fourier integral of $f(x)$. Be sure to indicate the Fourier integral coefficients.

(c) (7%) Similar to (b), derive the complex Fourier integral of $f(x)$. Also, indicate the complex Fourier integral coefficients specifically.

4. (15%) Find the solution for the following problem.

$u_{tt} = 8u_{xx} + xt$ for $-\infty < x < \infty, t > 0$,

$u(x,0) = x\cos(x), u_t(x,0) = \sin(x)$ for $-\infty < x < \infty$

(背面仍有題目,請繼續作答)

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Note that $\frac{\partial u}{\partial t} = u_t, \frac{\partial u}{\partial x} = u_x, \frac{\partial^2 u}{\partial t^2} = u_{tt}$

5. (15%) Use the inversion formula and the residue theorem to evaluate the inverse of the given Fourier transform

$$\hat{f}(\omega) = \frac{1}{\omega^2 + 1}$$

6. (20%) Use the inversion formula and the residue theorem to evaluate the inverse of the given Laplace transform

$$F(s) = \frac{1}{(s-2)^3}$$