

※ 考生請注意：本試題不可使用計算機

1. (20%) Solve the following initial value problems.

$$(a) y' = \frac{2y}{x} + x^2 e^x, y(2) = 0$$

$$(b) y' + 3x^2 y = x e^{-x^3}, y(0) = -1$$

2. (13%) Solve the following initial value problem.

$$y'' + 3.7y' = 0, y(-2) = 4, y'(-2) = 0$$

3. (17%) As we know, the Fourier series of $f(x) \forall x \in [L, -L]$ can be expressed as

$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right).$$

Now please answer the following questions.

(a) (3%) In what condition of $f(x)$, we should use Fourier integral to express $f(x)$ instead of Fourier series?

(b) (7%) Under the condition you express in (a), derive the Fourier integral of $f(x)$. Be sure to indicate the Fourier integral coefficients.

(c) (7%) Similar to (b), derive the complex Fourier integral of $f(x)$. Also, indicate the complex Fourier integral coefficients specifically.

4. (15%) Find the solution for the following problem.

$$u_{tt} = \theta u_{xx} + xt \text{ for } -\infty < x < \infty, t > 0.$$

$$u(x, 0) = x \cos(x), u_t(x, 0) = \sin(x) \text{ for } -\infty < x < \infty$$

(背面仍有題目, 請繼續作答)

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$$\text{Note that } \frac{\partial u}{\partial t} = u_t, \frac{\partial u}{\partial x} = u_x, \frac{\partial^2 u}{\partial t^2} = u_{tt}$$

5. (15%) Use the inversion formula and the residue theorem to evaluate the inverse of the given Fourier transform

$$\hat{f}(\omega) = \frac{1}{\omega^2 + 1}$$

6. (20%) Use the inversion formula and the residue theorem to evaluate the inverse of the given Laplace transform

$$F(s) = \frac{1}{(s-2)^3}$$