系所組別：工業與資訊管理學系甲組

考試科目：作業研究

考試日期：0224，節次：2

※ 考生請注意：本試題不可使用計算機

1. In playing a scissors-stone-paper (剪刀石頭布) game with your nephew, you know he will use scissors, stone, and paper with probabilities 0.3, 0.5, and 0.2, respectively. Suppose the loser gives one dollar to the winner for each game. What is your strategy for **losing** the most to him, and what is the expected value that you will lose for each game? (15%)

2. A professor asked you to collect the mean service time, mean inter-arrival time, the average number of customers in the system, and the average number of customers in the queue of a queueing system with only one server. After one month of observation, you found that the service times were different for each customer, with a mean of five minutes. The average numbers of customers in the system and queue were 5, and 4.5, respectively. However, you forgot to record the inter-arrival times of the customers. Can you calculate the average inter-arrival time of the customers from the data you already have? (15%)

3. The following questions are concerned with Markov chain.
   (a) What is the definition of Markov chain? (5%)
   (b) Write down the Chapman-Kolmogorov equations. (5%)
   (c) Prove that if state k is accessible from state j and state j is accessible from state i, then state k is accessible from state i. (10%)

(背面仍有題目,請繼續作答)

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4. (20%) Consider the following problem

$$\begin{aligned} \text{Maximize} \quad & Z = c_1x_1 + c_2x_2 + c_3x_3 \\ \text{subject to} \quad & 2x_1 + 3x_2 + x_3 \leq b_1 \\ & x_1 + x_2 + 2x_3 \leq b_2 \\ & x_1, \quad x_2, \quad x_3 \geq 0. \end{aligned}$$

The resulting final simplex tableau is as follows (where x_4 and x_5 are the slack varialbes for the functional constraints).

B.V.	x_1	x_2	x_3	x_4	x_5	RHS
Z	$\frac{7}{10}$	0	0	$\frac{4}{5}$	$\frac{3}{5}$	d
x_2	a_1	1	0	a_3	$-\frac{1}{5}$	3
x_3	a_2	0	1	a_4	$\frac{3}{5}$	1

Identify the value of $a_1, a_2, a_3, a_4, b_1, b_2, c_1, c_2, c_3$ and d.

5. A company makes two products, A and B. One unit of Product A requires 1.5 hours on machine 1, 2 hours on machine 2, and 1 hour on machine 3. One unit of Product B requires 0.5 hours on machine 1, 2 hours on machine 2, and 1.5 hours on machine 3. Each machine is availbe only 8 hours per day. The profit per unit sold is 15 and 10 for product A and B, respectively.

a) Formulate an integer programming for this problem. (5%)

b) Apply branch-and-bound algorithm to solve this problem. (15%)

6. (10%) Find all feasible solutions to the following set of constraints for a pure binay integer problem.

$$\begin{aligned} & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \geq 2 \\ & x_1 + x_2 - x_4 \geq 0 \\ & x_1 - 3x_5 + 3x_6 \geq 2 \\ & x_2 + x_4 + x_6 \leq 1 \\ & 3x_3 - x_5 + x_7 \leq 1 \end{aligned}$$