

國立交通大學 102 學年度碩士班考試入學試題

科目：統計學(4083)

考試日期：102 年 2 月 3 日 第 3 節

系所班別：統計學研究所

組別：統計所

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【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. Suppose X_1, X_2, \dots, X_n is a sample from an exponential pdf with the density function

$$f(x; \theta) = e^{-(x-\theta)}, x \geq \theta; \theta > 0.$$

(10%) (a) Find a sufficient statistic for θ .

(10%) (b) Find the maximum likelihood estimate for θ .

2. Let Y_1, \dots, Y_n be a random sample from a distribution with a density function $f(y; \theta) = 2y/\theta^2, 0 < y < \theta$.

(5%) (a) Is $\hat{\theta} = \frac{3}{2}\bar{Y}$ an unbiased estimator of θ ?

(15%) (b) Is the variance of $\hat{\theta}$ less than the Cramer-Rao lower bound for $f(y; \theta)$?

3. Let Y_1, \dots, Y_n be a random sample from a normal distribution $N(\mu, \sigma^2)$ with unknown μ and σ^2 .

(10%) (a) Find a uniformly most powerful test for testing $H_0 : \sigma^2 \leq \sigma_0^2$ versus $H_1 : \sigma^2 \geq \sigma_0^2$.

(10%)(b) Find a level $(1 - \alpha)$ confidence interval for σ^2 .

(10%)(c) Does there exist a uniformly most powerful test for testing

$H_0 : \sigma^2 = \sigma_0^2$ versus $H_1 : \sigma^2 \neq \sigma_0^2$?

4. An experiment takes a sample of size 1 from the Poisson probability model, $p(k) = e^{-\lambda} \lambda^k / k!, k = 0, 1, 2, \dots$ and wishes to test

$$H_0 : \lambda = 6 \text{ versus } H_1 : \lambda < 6$$

by rejecting H_0 if $k \leq 2$.

(10%) (a) What is the type I error?

(10%) (b) What is the power when $\lambda = 4$?

5. (10%) Explain what the p-value means in hypothesis testing.

(解釋 p-value 在統計檢定上的意義及用處)