國立交通大學 102 學年度碩士班考試入學試題

科目：機率論(4082)　　考試日期：102 年 2 月 3 日 第 2 節

系所班別：統計學研究所　組別：統計所　第 / 頁,共 / 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

1. Assume that the height of men in Taiwan is normally distributed with mean 65 inches and standard deviation 2 inches. The height of women is also normal, with mean 63 inches and standard deviation 1.5 inches. One man and one woman are selected at random. Let p be the probability that the woman selected is taller than the man selected.
   (a) (10%)Write p as a double integral.
   (b) (10%)Write p in terms of Φ, the cumulative distribution function of standard normal.

2. Suppose you buy 25 batteries and use them one at a time and replace the old one with a new one immediately when the old one failed. Assume that the life times of these batteries are random variables with mean 2 weeks and standard deviation 3 days.
   (a) (12%)What is the (approximate) probability that the batch of batteries will last more than a year?
   (b) (8%)What theorem and assumptions were you used to answer (a)?

3. Let $X_1, X_2, \cdots, X_n$ be i.i.d. Bernoulli trials with p the probability of success.
   (a) (5%)Find $P(X_1 = 1 \mid \sum_{i=1}^{n} X_i = r)$.
   (b) (15%)Find $Cov(X_i, X_j \mid \sum_{m=1}^{n} X_m = r)$ for $i \neq j$.

4. (20%)Let X be a *Gamma* (α, β) random variable where α is a positive integer. For any positive x, let Y be a *Poisson* (x/β) random variable. Show that $P(X \leq x) = P(Y \geq \alpha)$.

5. (20%)Let $X_1, X_2, \cdots$ be i.i.d. *uniform* (0, 1) random variables and let $X_{(n)} = \max_{1 \leq i \leq n} X_i$. Find the limiting distribution of $n(1 - X_{(n)})$ as $n \to \infty$.