國立交通大學102學年度碩士班考試入學試題

科目：離散數學(4041)　　考試日期:102年2月3日 第3節

系所班別：應用數學系　　組別：應數系乙組　　第 1 頁,共 1 頁

【不可使用計算機】＊作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符！！

每題要有完整的解釋才能得到全部的分數。

1. Let $\phi(m)$ denote the number of positive integers not exceeding m that are relatively prime to m. Prove that $\Sigma_{k|n}\phi(k) = n$. (15 points)

2. Use Pigeon Hole principle to prove that the decimal expansion of a positive rational number is repeating. (正分數可以化為循環小數) (15 points)

3. Find the number of different ways to obtain 300 dollars by collecting coins of values 1, 5, 10 and 50 dollars. (15 points)

4. Find the number of non-equivalent ways to color the corners of a square by 7 colors. (Two colorings of the corners of a square are equivalent if they can become the same coloring by using rotations or reflections of the square.) (15 points)

5. Let $X = \{1, 2, 3, \cdots, 13\}$. Find a collection $\mathcal{B}$ of 4-subsets of X such that any 2-subset of X is contained in exactly one 4-subset of $\mathcal{B}$. (15 points)

6. A tree is a simple connected graph without cycles.
   Let $D = (d_1, d_2, \cdots, d_n)$ be a sequence of n positive integers such that the sum of these n integers is equal to 2(n-1). Prove or disprove that there exists a tree T with degree sequence D. (15 points)

7. A Hadamard matrix of order n is an n by n matrix H with entries +1 and -1, such that $H H^T = nI$ where I is the identity matrix of order n. Find a Hadamard matrix of order 8. (10 points)