國立交通大學 102 學年度碩士班考試入學試題

科目：應用數學(4021)　　考試日期：102 年 2 月 3 日 第 1 節

系所班別：電子物理學系　組別：電物系乙組、甲組　　第 1 頁，共 2 頁

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1. Find the eigenvalues and eigenvectors of matrix A, where

$$A = \begin{pmatrix} 0 & i & 0 \\ -i & 1 & -i \\ 0 & i & 0 \end{pmatrix}$$

What property do you expect for the eigenvectors, and why?

(10 points)

2. One-electron Schrödinger equation can be put into Lippmann-Schwinger form:

$$\psi(x) = \int_{-L}^{L} dx_1 \int_{-L}^{L} dx_2 G(x, x_1) V(x_1, x_2) \psi(x_2),$$

which is an integral equation representing an electron in a box (-L,L). Suppose that the wave function, the potential, and the Green's function can be expanded by plane waves basis, respectively, i.e.,

$$\psi(x) = \sum_n \psi_n e^{ik_n x},$$

$$G(x, x') = \sum_{n,n'} e^{ik_n x} G_{n,n'} e^{-ik_{n'} x'},$$

$$V(x, x') = \sum_{n,n'} e^{ik_n x} V_{n,n'} e^{-ik_{n'} x'}, \text{ where } k_n = \frac{\pi}{L} n \text{ and n is an integer.}$$

Show that the Lippmann-Schwinger equation can be transform to a matrix form:

$$\sum_{n'} A_{n,n'} \psi_{n'} = \psi_n.$$

(a) What is the form of $A_{n,n'}$ in terms of $G_{n,n'}$ and $V_{n,n'}$?

(10 points)

(b) Describe how you can solve the wave function $\psi(x)$ from the message of (a)?

(10 points)

3.

(a) If $f(z)$ is analytic inside and on a closed contour C. If N is the number of zeros of $f(z)$ inside C. Calculate the value of

$$\oint_C \frac{f'(z)}{f(z)} dz$$

(10 points)

(b) Using complex integral techniques to evaluate the following integral,

$$\int_0^{2\pi} \frac{dx}{a + \cos x}$$

(10 oints)

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4. Solve the differential equation: $y' = y(xy^3 - 1)$.

(15 points)

5. Solve $4y'' + 36y = \csc 3x$.

(10 points)

6. Solve $y'' - (1+x)y = 0$ by using power series solutions. Find at least 4 terms of each solution.

(15 points)

7. One definition of the gamma function is given by the improper integral

$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt, \alpha > 0$. Show that

(a) $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$

(b) $L[t^\alpha] = \dfrac{\Gamma(\alpha+1)}{s^{\alpha+1}}$.

Use the fact that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ to find the Laplace transform of

(c) $f(t) = t^{-1/2}$, and

(d) $f(t) = t^{3/2}$.

(10 points)