

1. 21% (a) Solve for $\frac{dy}{dx} = \frac{y}{x} + y^3$? (7%) (b) Solve for $\frac{d^2y}{dt^2} + y = \cos t$? (7%)
 (c) Solve for $x''(t) + x(t) = \begin{cases} 0, & 0 \leq t < 1 \\ t, & 1 \leq t < \infty \end{cases}$ $x(0) = 0$ $x'(0) = 0$ (7%)
2. 15% If $u = \exp(\sin(x + y + z))$, $\vec{s} = 2\vec{i} + \vec{j} + \vec{k}$, evaluate (a) $\vec{\nabla}u$ (5%) (b) ∇^2u (5%)
 (c) $\frac{du}{ds}$ at $(2, 0, 1)$? (5%)
3. 10% Explain how to use Fourier transform to solve the equation: $y''(x) + y(x) = \frac{1}{x^2 + 1}$? You need only to show the steps and list the formula required to get the solution. You don't need to evaluate the integral.
4. 10% (a) Find the Fourier integral of $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$? (5%) (b) $\int_0^\infty \frac{\sin(\omega)}{\omega} d\omega = ?$ (5%)
5. 9% Find the eigenvalues and the corresponding unit eigenvectors of the matrix, $\begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$? (9%)
6. 5% Use the Gram-Schmidt process to find an orthogonal basis of $\langle 1, 4, 0 \rangle, \langle 2, -5, 0 \rangle$ spanning in \mathbb{R}^3 ? (5%)
7. 20% Find $[A_R]$, the reduced row echelon form of the matrix, $[A] = \begin{bmatrix} 2 & -3 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \\ 1 & -3 & 1 & 0 \end{bmatrix}$? (8%) What is the rank of $[A]$? (2%) If there is a linear transformation: $[A]: \mathbb{R}^n \rightarrow \mathbb{R}^n$. Find its null space? (8%) What is its dimension? (2%)
8. 10% (a) Find the walks of length 4 from v_1 to v_4 in the graph? (5%) (b) Find the total number of spanning trees in the graph? (5%)

