

【可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. A third order ordinary differential equation, $y''' - y'' - 4y' + 4y = 0$, has a general solution, $y = c_1 e^x + A(x) + B(x)$.

- (1) Please specify the explicit functional forms of $A(x)$ and $B(x)$. (10%)
- (2) If the initial conditions are $y(0) = 1, y'(0) = 0, y''(0) = -1$, Please define the particular solution. (10%)

2. $\lambda = 3$ is one of eigenvalues of the following matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & -2 \\ -5 & 3 & 8 \end{bmatrix}$$

- (1) Please find the other eigenvalues of the A matrix. (10%)
 - (2) Please define eigenvectors associated to the eigenvalues. (10%)
3. A velocity field of water flow and a parabolic cylinder are given by $\mathbf{V} = [3z^2, 6, 6xz]$ and $S: y = x^2, 0 \leq x \leq 2, 0 \leq z \leq 3$ of which the unit is cm.

- (1) Give a representation of the surface S. (5%)
 - (2) Find the normal vector at the point (1, 1, 1) (5%)
 - (3) Compute the mass flux through the surface S. The density is 1g/cm^3 . (5%)
4. A periodic function is given by $f(x) = x^2, -1 < x < 1$ of period $p = 2L = 2$.
- (1) Is the function odd or even? (2%)
 - (2) Find its Fourier series. (13%)
 - (3) Using the result of (2) show that $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots = \frac{\pi^2}{12}$. (5%)

Note: Fourier series

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L}, a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx, a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx$$

5. Evaluate the following improper integral using residue integration. (15%)

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$$

6. A velocity field of a fluid flow is given by $\mathbf{V} = xi - yj$.

- (1) Is the fluid is incompressible? (Show the detail and Explain) (5%)
- (2) Is the flow irrotational? (Show the detail and Explain) (5%)