

國立交通大學 102 學年度碩士班考試入學試題

科目：工程數學(3051)

考試日期：102 年 2 月 4 日 第 1 節

系所班別：土木工程學系

組別：土木系甲組一般生

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【可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. Solve the following equation in t -domain,

$$\mathbf{M}\mathbf{x}'' + \mathbf{C}\mathbf{x}' + \mathbf{K}\mathbf{x} = \mathbf{0},$$

given that $\mathbf{x}(0) = \mathbf{0}$ and $\mathbf{x}'(0) = (1, 0, 0)^T$, where $' = d/dt$ and

$$\mathbf{M} = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{K} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & 2 \\ 0 & 2 & 2 \end{bmatrix}. \quad (20\%)$$

2. Analytically find the general solutions for

$$x^2 y'' + (x^3 - 3x)y' - 2x^2 y = 0$$

where $' = d/dx$. (30%)

$$3. \quad \mathbf{A} = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

(a) Find \mathbf{A}^{-1} using the method of Gauss-Jordan elimination. (8%)

(b) Find the eigenvalues of \mathbf{A}^{-1} . (8%)

(c) Find the eigenvalues of \mathbf{A}^5 . (7%)

(d) If $\mathbf{D} = \mathbf{P}^{-1} \mathbf{A} \mathbf{P}$ is a diagonal matrix, find the matrix \mathbf{P} . (7%)

4. Given a function $f = \ln(x^2 + y^2)$, find the directional derivative of f at point $P: (3, 0)$ in the direction of $\mathbf{a} = [1, -1]$. (10%)

5. Given a vector field $\mathbf{F} = [5x^2, 0, 3y]$, evaluate the surface integral

$$\iint_S \mathbf{F} \cdot \mathbf{n} dA$$

where the surface S is the portion of the plane $x+y+z=1$ in the first octant. (10%)

(Note:

$$\iint_S \mathbf{F} \cdot \mathbf{n} dA = \iint_R \mathbf{F}[\mathbf{r}(u, v)] \cdot \mathbf{N}(u, v) du dv$$

where $\mathbf{N} = \mathbf{r}_u \times \mathbf{r}_v$)