1. (a) (5%) prove that

$$\nabla^2(fg) = f\nabla^2 g + 2\nabla f \cdot \nabla g + g\nabla^2 f$$

Then (b) (5%) show that

$$\nabla^2\left(\frac{-1}{4\pi r}\right) = \delta(r)$$

(c) (10%) an electrostatic potential is

$$\varphi(\mathbf{r}) = \frac{Z}{4\pi\varepsilon_0}\frac{e^{-ar}}{r}$$

Reconstruct the electrical charge distribution that will produce this potential. Note that $\varphi(r)$ vanishing exponentially for large $r$, showing that the net charge is zero. (hint: $\nabla^2\varphi(\mathbf{r}) = -\frac{\rho(\mathbf{r})}{\varepsilon_0}$)

2. (15%) a sphere of radius $R$ carries a charge density $\rho(\mathbf{R}) = kR$. Find the energy of the configuration.

3. (11%) a cylindrical bar magnet of radius b and length L has a uniform magnetization $\mathbf{M} = \mathbf{e}_z M_0$. Please describe the magnetic flux density $\mathbf{B}$ at an arbitrary distant point.

4. (12%) Consider a light beam $\mathbf{E}_i$ incident obliquely on a mirror (as x-y plane) with angle of incidence $\theta_i$, please describe the reflected light $\mathbf{E}_r$ in cases of (a) perpendicular polarization input and (b) parallel polarization input, respectively.

5. (12%) The microwave oven and induction cooker are two widely used household appliances for food heating. What are the operating principles of (a) microwave oven and (b) induction cooker?

6. (10%) A transmission line of characteristic impedance $R_0 = 50\ (\Omega)$ is to be matched to a load impedance $Z_L = 60 + j10(\Omega)$ through a length $L'$ of another transmission line of characteristic impedance $R_0'$ Find the required $R_0'$ for matching.

7. (15%) A parallel-plate waveguide made of two perfectly conducting infinite planes spaced 3 (cm) apart in air operates at a frequency 10 (GHz). Please:

   (a) (5%) Explain what is meant by a "cutoff frequency" of a waveguide

   (b) (10%) Find the maximum time-average power that can propagated per unit width of the guide without a voltage breakdown for the $TM_1$ mode.

8. (5%) According to the following radiation patterns, what is the "three" most important parameter to represent the characteristic of an Antenna?