問題 1 至 10 中總共有 20 個空格((A1),(A2),...,(A19),(A20)) 。請根據題意，將適當的數字、向量、矩陣、函數、集合、符號或文字等作答於答案卷。例如：(A1)=20, (A2)=5x+6 等。

1. (5%)The solution $f(t)$ to the differential equation $tf' - (t+2)f = -2t^2 - 2t$ can be expressed as the linear combination of the following two terms: t and (A1).　　(A1) = ? (5 points)

2. (5%)If $L[f(t)] = \frac{s}{s^4 + a^4}$ with $a > 0$, then $f(t) =$ (A2).　　(A2) = ? (5 points)

3. (5%)The solution to the initial value problem $f'' - 3f' + 2f = 4t + e^{3t}$, $f(0) = -f'(0) = 1$ can be written as $f(t) =$ (A3).　　(A3) = ? (5 points)

4. (10%)A partial differential equation of the form $\nabla^2 f(\bar{r}) = h(\bar{r})$ is called the Poisson's equation, where $f(\bar{r})$ is the unknown "potential" function to be determined and $h(\bar{r})$ is a known "source" function (all in the spherical coordinate system). The most common boundary condition applied to this equation is that $f(\bar{r})$ is zero as $|\bar{r}| \to \infty$. Provided that $g(\bar{r}) = \frac{1}{4\pi|\bar{r}|}$ is the solution to the equation $\nabla^2 g(\bar{r}) = \delta(\bar{r})$, then the solution to the equation $\nabla^2 G(\bar{r}) = \delta(\bar{r} - \bar{r}_0)$ can be expressed as $G(\bar{r}) =$ (A4). Besides, since Poisson's equation is linear in both the potential and the source terms, the solution to its most general form $\nabla^2 f(\bar{r}) = h(\bar{r})$ could be written as $f(\bar{r}) =$ (A5).　　(A4) = ? (3 points).　　(A5) = ? (7 points)

5. (10%)The solution of the Bessel's function of first kind with order 1/2 is expressed as $J_{1/2}(x) = \left(\frac{x}{2}\right)^{\frac{1}{2}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\Gamma\left(\frac{3}{2}+k\right)} \left(\frac{x}{2}\right)^{2k}$, where $\Gamma\left(\frac{3}{2}+k\right) = \int_0^{\infty} t^{\frac{1}{2}+k} e^{-t} dt$ and $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. The solution can be rewritten as $J_{1/2}(x) =$ (A6) $\sin x$.　　(A6) = ? (10 points)

6. (5%)The solution of the Bernoulli's equation $y' - \frac{6}{17} y \sin x = \frac{5}{32} x^4 y^2, y(0) = 1$, $y(x)$, has a singular point at $x =$ (A7).(Hint: find $y(x)$ when $y >> 1$)　　(A7) = ? (5 points)

7. (10%)The solution of the diffusion equation $Dy'' - vy' - \alpha y = M\delta(x)$ is expressed as $y(x) =$ (A8) $\times\, e^{(A9)x}$, when $x > 0$.　　(A8) = ? (5 points)　　(A9) = ? (5 points)

8. (20%) Given two 3×3 matrices A and B, assume they can be diagonalized by the same invertible matrix P such that $M_1 = P^{-1}AP$ and $M_2 = P^{-1}BP$ are diagonal matrices. Let $A = \begin{bmatrix} 3 & -2 & 0 \\ 1 & 3 & -3 \\ 1 & 4 & -4 \end{bmatrix}$, $M_1 = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$ with $\lambda_1 \geq \lambda_2 \geq \lambda_3$, and $M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$. Let $M_1 =$ (A10), $P =$ (A11), $\det(B) =$ (A12), $AB - BA =$ (A13)

(A10) = ? (5 points)　　(A11) = ? (5 points)　　(A12) = ? (5 points)　　(A13) = ? (5 points)

見背面

9. (5%) Let u and v be two vectors with norms $\|u\| = 4$ and $\|v\| = 7$, respectively, in some inner product space. Let $\langle 2u+v, 2u-v \rangle$ = (A14)

(A14) =? (5 points)

10. (25%) The field Z_2 consists of two elements 0 and 1 with the operations of addition(+) and multiplication(•) defined by the following equations.

$0+0=0,\quad 0+1=1+0=1,\quad 1+1=0;$

$0\bullet 0=0,\quad 0\bullet 1=1\bullet 0=0,\quad 1\bullet 1=1.$

Let V be the 3-tuple vector space of $(Z_2)^3$, i.e. if $v \in V$ then $v=\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ in which $v_i \in Z_2$ for $i=1,2,3$.

Let M be the 3 by 3 matrices defined on Z_2, i.e. if $T \in M$ then $T=\begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}$ in which $t_{ij} \in Z_2$ for $i=1,2,3$ and $j=1,2,3$

(a) The zero vector in V is (A15).

(A15) =? (2 points)

(b) Let $v \in V$ and $v=\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, its additive inverse is (A16).

(A16) =? (3 points)

(c) Let $T \in M$ and $T=\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and T^{-1} = (A17).

(A17) =? (5 points)

(d) Let $U \in M$ and $U=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ the eigenvalue set of U is (A18); the associated eigenvector set is (A19).

(A18) =? (5 points)　　(A19) =? (5 points)

(e) Let L be the linear transformation from V to V, and $L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right)=\begin{bmatrix} a \\ a+b \\ a+b+c \end{bmatrix}$.

Assume $B=\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix},\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}$ is the basis of V. Let $[L]_B$ = (A20) be the matrix representation of L with respective to B.

(A20) =? (5 points)

試題隨卷繳回