1. Solve the linear differential system (20%)

$$\frac{dx}{dt}+2x+\frac{dy}{dt}+6y=e^{-t}$$

$$2\frac{dx}{dt}+3x+3\frac{dy}{dt}+8y=t^2$$

2. Solve the differential equation following heat conduction problem (15%)

$$\frac{d^3y}{dt^3}+3\frac{d^2y}{dt^2}+2\frac{dy}{dt}=\delta(t-1)$$

with the boundary conditions

$$\frac{d^2y}{dt^2}(0)=\frac{dy}{dt}(0)=0 \quad \text{and} \quad y(2)=1$$

3. For matrix $\mathbf{A}=\begin{bmatrix}2 & -1 & 5 & 0\\ 3 & 0 & -2 & 0\\ 1 & 4 & 0 & 0\\ 0 & 0 & 0 & 2\end{bmatrix}$,

(a) det **A** = ? (2%) (b) What is the rank of **A** ? (2%) (c) What is the inverse of **A**?(6%)

4. Let **A** be a symmetric matrix with real entries. Show that the eigenvalues of **A** are real.(10%)

5. What is the Gauss divergence theorem?(5%) If $\mathbf{F} = xy\,\mathbf{i} + y^2z\,\mathbf{j} + z^3\,\mathbf{k}$, evaluate $\iint_S (\mathbf{F}\cdot\mathbf{n}\,dS)$, where S is the unit cube defined by $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$. (5%)

6. (20%) 求解 (a) 2^i ; (b) $\int_0^\infty \frac{x^{1/3}}{x(x^2+1)}dx$

7. (15%) 求解 $\begin{cases}\dfrac{\partial u}{\partial t}=k\dfrac{\partial^2 u}{\partial x^2}; & \text{for } 0<x<\infty, t>0\\ u(0,t) = 0; & \text{for } t\ge 0\\ u(x,0)=f(x); & \text{for } 0<x<\infty\end{cases}$