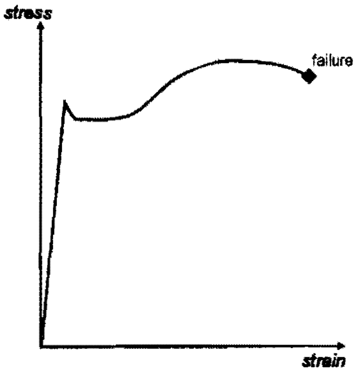
Problem 1. (25%)

(1) Please provide us ***Young's modulus***, typical ***yielding strength***, and ***density*** of ***aluminum alloys*** and ***low carbon steels***. (4 %)

(2) The following figure is the typical engineering stress-strain curves for common low carbon steels. Please tell us how to find the following material properties from the curve: ***Young's modulus***, ***yielding strength***, ***modulus of resilience***, and ***modulus of toughness***. In addition, please also draw the corresponding ***true stress-strain curve***. (8 %)



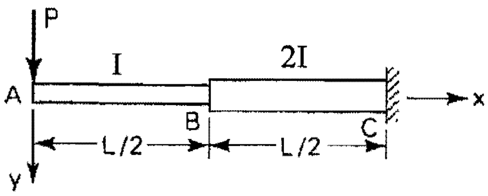
(3) Bulk modulus K of an isotropic material is defined as $K \equiv -p/(\Delta V/V)$, where ΔV is the change in a volume V produced by hydrostatic pressure p. Please show that $K = \dfrac{E}{3(1-2\nu)}$. (E and ν are Young's modulus and Poisson's ratio.) (13 %)

Problem 2. (25 %)

(1) Please show that for a uniform beam (length L, inertia I, Young's modulus E) under pure bending (bending moment M), the strain energy can be expressed as $U = \int_0^L \dfrac{M^2}{2EI} dx$. (5 %)

(2) Please state the principle of minimum potential energy and explain its physical meaning. (5 %)

(3) Consider the stepped cantilever beam shown below (with moment of inertia I and $2I$), please determine the vertical deflection and slope at point A by Castigliano's theorem. (15 %)



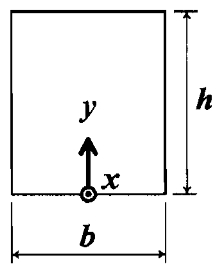
Problem 3. (25%)

Consider a slender beam that has a rectangular cross section as shown below. The beam is nonhomogeneous along the transverse direction such that the Young's modulus and the coefficient of thermal expansion are $E(y)=E_0\left(1+\frac{y}{h}\right)$ and $\alpha(y)=\alpha_0\left(2-\frac{y}{h}\right)$, respectively.

Given that the beam is stress- and deformation-free at temperature 0 and is subjected to a uniform temperature change from 0 to *T*, answer the following questions:

(1) Write the functional form of the strain distribution $\varepsilon_x(y)$ without determining the constants. (5%)

(2) Determine the stress distribution $\sigma_x(y)$ and the bending radius of curvature of the beam at temperature *T*. (20%)



Problem 4. (25%)

Consider an elastic-perfectly-plastic circular shaft of length *L* and radius *c* subjected to a torque *T*. Shear modulus and shear yielding strength of the shaft are *G* and τ_Y, respectively.

(1) Given the torque $T=2c^3\tau_Y$, determine the shear stress distribution $\tau(r)$ across a section of the shaft, *r* being the distance from the axis of the shaft. (15%)

(2) Determine the permanent angle of twist after the torque is released. (10%)