※ 考生請注意：本試題可使用計算機，並限「考選部核定之國家考試電子計算器」機型

1. A cylinder/piston consists of an ideal gas with mass m and constant specific heats. Its specific heat ratio is k and gas constant is R. If kinetic and potential energy changes are negligible, show that for a reversible adiabatic process (20%)

   (a) $W = \dfrac{mR(T_2 - T_1)}{1-k}$

   (b) $\dfrac{T_2}{T_1} = \left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

2. An engineer wishes to design an industrial process which requires a steady 0.5 kg/s of air at 200 kPa, 300 K. This air is to be the exhaust from a specially designed turbine with inlet pressure 400 kPa. The heat transfer comes from a source at a temperature 100°C higher than the turbine inlet temperature. The turbine process may be assumed to be reversible and polytropic, with polytropic exponent $n = 1.2$, and the changes in kinetic and potential energy are negligible. This air is an ideal gas, with constant specific heat, $C_p = 1.004$ kJ/kg-K, and $R = 0.287$ kJ/kg-K. Verify that the engineer can meet the goal and this process can take place in accordance with the principle of the increase of entropy. (30%)

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3. Consider an ideal air-standard Brayton cycle with minimum and maximum temperatures of 300 K and 1650 K, respectively. The pressure at the compressor inlet is 1 bar. Ignore kinetic and potential energy effects. The pressure ratio is that which maximizes the net work developed by the cycle per unit mass of air flow. On a cold air-standard basis with specific heats for the air, $c_v = 0.718\ \text{kJ/kg-K}$ and $c_p = 1.005\ \text{kJ/kg-K}$,

(a) derive the pressure ratio which maximizes the net work developed by the cycle per unit mass of air flow and calculate its value for the present case,

(b) calculate the compressor work and the turbine work per unit mass of air flow, each in kJ/kg, and the thermal efficiency of the cycle. (25 %)

4. The specific entropy can be regarded as a function of the form $s = s(t, v)$. Similarly, the specific internal energy can be regarded as a function of the form $u = u(t, v)$.

(a) Show that $\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p$.

(b) Using the virial equation of state, $pv = RT\left[1 + \frac{B(T)}{v} + \frac{C(T)}{v^2} + \frac{D(T)}{v^3} + \cdots\right]$, calculate $\lim_{v\to\infty}\left(\frac{\partial u}{\partial v}\right)_T$. (25 %)