

系所組別：環境工程學系丙組

考試科目：微積分

考試日期：0225·節次：3

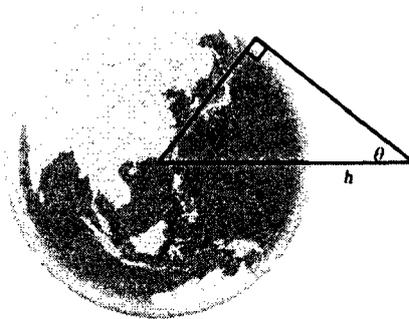
1. Please find the derivative of the following functions.(18%)

(1) $y = 3\sec^2(\pi x - 1)$ (6%)

(2) $y = \log_5 \frac{x\sqrt{x-1}}{2}$ (6%)

(3) $y = \ln\left(\frac{1+e^x}{1-e^x}\right)$ (6%)

2. When satellites observe Earth, some satellites have sensors that are capable to measure the angle θ as shown in the figure. Let h denote the distance between satellite and Earth's surface and let r represent Earth's radius ($r = 6378$ km). Please calculate the rate at which h is changing with respect to θ when $\theta = 30^\circ$. (15%)



3. Please answer the following questions. (22%)

(1) Please find the **extreme** and **reflection points** of $f(x) = x^2 e^{-x}$ (6%)

(2) Please find an equation of **tangent line** to the graph of $y = (\ln x)^{\cos x}$ at $(e, 1)$ (6%)

(3) Let $0 \leq x \leq 2$, please evaluate the definite integral for the **surface area** generated by revolving $y = 1 - \frac{x^2}{4}$ about the y -axis. (10%)

4. Please evaluate the integral of the following functions. (25%)

(1) $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$ (5%)

(2) $\int \frac{1}{\sec x \tan x} dx$ (5%)

(3) $\int \frac{x}{16x^4 - 1} dx$ (5%)

(4) $\int_0^1 e^x \sin x dx$ (5%)

(5) $\int_0^\pi \int_0^{\sin x} (1 + \cos x) dy dx$ (5%)

5. The maximum volume of an ellipsoid. (20%)

(1) Please calculate the volume of an ellipsoid given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (10%)

(2) For a fixed sum $a + b + c = k$, please show that the ellipsoid of maximum volume is a sphere. (10%)