

I. Please solve the following differential equations: (5 points for each one)

A. $y'' + 4y' + 13y = \delta(t - \pi) + \delta(t - 3\pi)$ with $y(0) = 1, y'(0) = 1$

B. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^x \cos 2x$ C. $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = \ln x^2$ D. $y'' - y = \frac{2e^x}{e^x + e^{-x}}$

II. Please find the eigenvalues and eigenfunctions for the following boundary-value problems. (5 point for each)

A. $y'' + \lambda y = 0, \quad y'(0) = 0, \quad y'(\pi) = 0$

B. $x^2 y'' + xy' + \lambda y = 0, \quad y(1) = 0, \quad y(e^\pi) = 0$

III. The drag force on the particle, F_d , is proportional to particle velocity with constant coefficient k , that is, $F_d = -kU$, where U is the particle velocity.

(A) If the particle is injected horizontally into still air, please find the maximum horizontal traveling distance. (10 points)

(B) If the particle is injected upward into still air, what is the maximum upward traveling distance. (10 points)

IV. Please solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ with the following conditions: (10 points for each one)

A. $\begin{cases} u(x, 0) = \sin x, & 0 < x < \pi \\ t > 0, & u(0, t) = 5, \quad u(\pi, t) = 5 \end{cases}$

B. $\begin{cases} u(x, 0) = 100, & 0 < x \\ t > 0, & u(0, t) = 25, \quad u(\infty, t) = 100 \end{cases}$

C. $t > 0, u(x, 0) = \begin{cases} 0, & x < 0 \\ e^{-x}, & x > 0 \end{cases}$

V. Please write down the computation equations explicitly if the fourth order Runge-Kutta method is used for the differential equation $y'' - y(y')^2 + 5xy^3 = x$ with initial conditions $y(0) = 1$ and $y'(0) = 5$. (10 points)

VI. Please derive the condition for stable solution if the explicit finite difference method is used to

solve the partial differential equation $\frac{\partial^2 T}{\partial t^2} = c^2 \frac{\partial^2 T}{\partial x^2}$. (10 points)