國立臺灣大學 102 學年度碩士班招生考試試題

1. Please show the probability density function, the support, as well as the mean and variance, of the following distributions

(1). A binomial distribution with n trials and the probability of success p (5%)

(2). A Poisson distribution with intensity λ (5%)

(3). A chi-square (χ^2) distribution with n degrees of freedom (10%)

2. Let $X \sim N(\mu, \sigma^2)$, and $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from X. Let $\bar{X}$ and S^2 be the mean and variance of the sample, respectively. Let $\bar{x}$ and s^2 be the **observed** mean and variance of the sample, respectively. Please

(1). calculate the covariance of $\bar{X}$ and S^2 (5%)

(2). derive the sampling distribution of $\bar{X}$ (5%)

(3). derive the sampling distribution of S^2 (5%)

(4). derive the sampling distribution of $\frac{\bar{X}-\mu}{\sigma_{\bar{X}}}$, where $\sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}$ (5%)

(5). derive the sampling distribution of $\frac{\bar{X}-\mu}{S_{\bar{X}}}$, where $S_{\bar{X}}^2 = \frac{S^2}{n}$ (5%)

(6). construct a test for testing H_0: $\mu = \mu_0$ vs. H_A: $\mu \neq \mu_0$ with a significance level of α when σ^2 is **known** (5%)

(7). construct a test for testing H_0: $\mu = \mu_0$ vs. H_A: $\mu \neq \mu_0$ with a significance level of α when σ^2 is **unknown** (5%)

3. Let X be a random variable with a finite mean and variance, and $X_1, X_2, \ldots, X_n$ be a random sample of size n drawn from X. Let $\bar{X}$ be the mean of the sample. Please derive the **limiting distribution** of $\frac{\bar{X}-\mu}{\sigma_{\bar{X}}}$, as $n \to \infty$, and please briefly explain why (5%)

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4. Let X be a t distribution with n degrees of freedom. Please find the **limiting distribution** of X as $n \to \infty$, and please briefly explain why (5%)

5. Please briefly explain what the **Gauss-Markov theorem** is in linear regression analysis (15%)

6. Let the model of a complete randomized design be $Y_{ij} = \mu_i + \varepsilon_{ij}$, with $i = 1, 2, \ldots, a$, $j = 1, 2, \ldots, n$, and $\varepsilon_{ij} \sim N(0, \sigma^2)$. Please

   (1). state the null and alternative hypotheses (5%)

   (2). find the best estimators for μ_i (5%)

   (3). derive the expected values of ***MSTreat*** and ***MSE***, which are the mean squares of the treatments and errors, respectively, and the sampling distribution of $\frac{MSTreat}{MSE}$ under the null hypothesis (10%)