國立臺灣大學 102 學年度碩士班招生考試試題

1. (15%) Let X and Y be independent standard normal random variables. Find the distribution of $2XY/\sqrt{X^2+Y^2}$.

2. (15%) Let $U_1, \dots, U_n, \dots$ be i.i.d. Uniform(0,1) random variables and X have distribution

$$P(X=x) = \frac{1}{(e-1)x!} I_{\{1,2,\dots,\}}(x).$$

Find the distribution of $Z = \min\{U_1, \dots, U_X\}$.

3. (15%) Let $X_1, \dots, X_n$ be a random sample from a population with a probability density function

$$f(x|\theta_0) = \theta_0 x^{\theta_0 - 1} I_{(0,1)}(x),$$

where $0 < \theta_0 < \infty$. Find the uniformly minimum variance unbiased estimator of θ_0.

4. (10%) (15%) Let $X_1, \dots X_n$ be a random sample from $N(\theta, \sigma^2)$. Find an unbiased size α test for the hypotheses $H_0 : \theta_1 \leq \theta \leq \theta_2$ versus $H_A : \theta < \theta_1$ or $\theta > \theta_2$ and show that the given test is unbiased.

5. (15%) Let $X_1, \dots, X_n$ be a random sample from $Poisson(\lambda)$ and λ have a $Gamma(\alpha, \beta)$ distribution, where α and β are known positive constants. Find the Bayes estimator of λ under the squared error loss function.

6. (10%) (5%) Let $X_i \sim$ Binomial(n_i, p_i), $i = 1, \dots, m$, be independent. Derive the likelihood ratio test for $H_0 : p_1 = \dots = p_m$ versus $H_A : p_i \neq p_j$ for some $i \neq j$. What is the large sample distribution of the test statistic?

試題隨卷繳回