1. (34%)

(a) (6%) Evaluate $\int_0^1 x^{0.7} \ln x dx$.

(b) (6%) It is known that $\int_0^x g(x,y)dy = \int_0^x \int_0^z f(y)dydz$, where $f(y)$ is a given known function. Find $g(x,y)$.

(c) (8%) Given a matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find its eigenvalues and the associated eigenvectors. Whether or not the set of the three eigenvectors of A that you have found must always be mutually orthogonal to each other?

(d) (14%) Let $\tilde{v}(x,y,x) = (xy-1)\tilde{i} - xz\tilde{j} + (2-yz)\tilde{k}$, where $(\tilde{i},\tilde{j},\tilde{k})$ constitutes a set of orthogonal unit basis. Find a vector function $\tilde{w}(x,y,x)$ such that $\tilde{v} = \nabla \times \tilde{w}$. Is $\tilde{w}$ unique ?

2. (32%)

Let $y' = dy/dx$, $y'' = d^2y/dx^2$. Find the solutions of the following ordinary differential equations:

(a) (10%) $y''+5y'+6y=0$, $y(0)=1$, $y'(0)=2$.

(b) (12%) $x^2y''+xy'+y=0$ (for $x>0$), $y(1)=1$, $y'(1)=-1$.

(c) (10%) $\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, $y_1(0)=2$, $y_2(0)=3$.

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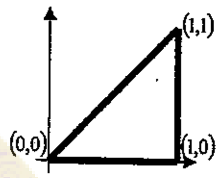
3. (14%)

(a) (6%) Consider a scalar function $\phi(x,y)=x^2+2xy$. Find the directional derivative of ϕ at the point $(1,1)$ in the direction $\mathbf{v}=(3,4)$.

(b) (8%) Evaluate the following integral around the curve C, traversed counterclockwise.

$$I=\oint_C[(2x+y)dx+2xdy]$$

where C is the boundary of the triangle with vertices (0,0), (1,0), and (1,1)



4. (20%)

Find the solution $u(x,t)$ of the following initial-boundary-valued problem

$$\begin{cases}\dfrac{\partial^2 u}{\partial x^2}=\dfrac{\partial^2 u}{\partial t^2}+4u; \quad 0<x<\pi, t>0\\ u(0,t)=0, \quad u(\pi,t)=0; \quad t>0\\ u(x,0)=\sin x\cdot\cos x, \quad \dfrac{\partial u}{\partial t}(x,0)=-5\sin 3x; \quad 0<x<\pi\end{cases}$$

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