1. **(14%)** An incompressible viscous fluid of density ρ and viscosity μ is pumped into a two-dimensional duct of uniform separation H by a constant streamwise pressure gradient, $\partial P/\partial x$. The flow evolves from a uniform entrance velocity, U, at x=0 to its fully-developed parabolic profile, $u(y)=Ay^2+By+C$, at $x=L_1$. The flow profile remains unchanged through another section of equal length, from $x=L_1$ to $x=L_2$ with $L_2=L_1$.

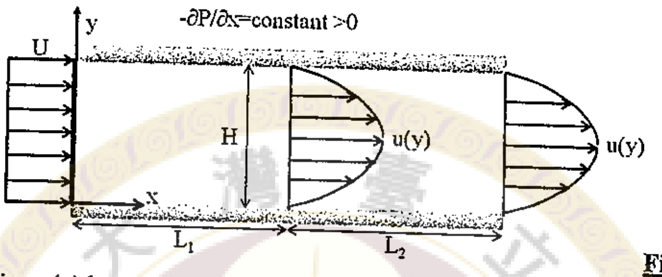

Fig. 1

3% (1) Determine u(y) by considering mass conservation and proper boundary conditions.

8% (2) Use control voloume analysis (integral momentum equation) to estimate the total wall friction force on the two duct walls, F_1, for $0 \le x \le L_1$.

3% (3) Let F_2 denote the total wall friction force in the downstream section, $L_1 \le x \le L_2$. Is $F_2>F_1$, $F_2=F_1$, or $F_2<F_1$? Why?

2. **(21%)** A self-stirring mug has a small upright fan at its base. When operating at full fan speed, ω, the contained incompressible liquid develops steady motion forming a dimple near the central free surface and a flat surface near the outer rim. Define a cylindrical cooridnates system at the base center. The fluid motion near the free surface can be approximated by an **inviscid** *Rankine vortex model* that describes the liquid tangential velocity in a composite manner, in an inner rigime I and an outer rigime II, as

$$u_\theta(r)=\begin{cases} r\omega, & \text{for } r \le R \text{ in regime I} \\ R^2\omega/r, & \text{for } r > R \text{ in regime II} \end{cases}$$

and $u_r \approx 0$ and $u_z \approx 0$. Here, R is comparable to the length of fan leaf. Some useful expressions in cylindrical coordinates system are given below and Problem 4.

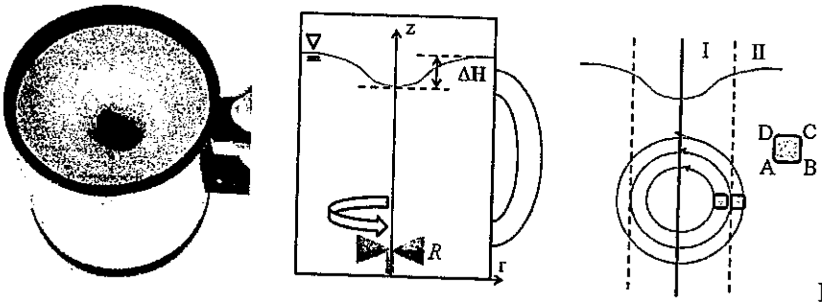

Fig. 2

4% (1) Calculate vorticity in both regimes.

4% (2) Draw how a square fluid element ABCD in the right sketch in Fig. 2 would move or deform in regime I and II, respectively. Explain your results.

13% (3) Solve for the pressure profile in both regimes. Show that the surface at r=0 is at a depth of $\Delta H = R^2\omega^2/g$ below the flat surface in the outer regime II, with g denoting gravity.

Gradient operator: $\vec{\nabla} = \frac{\partial}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\vec{e}_\theta + \frac{\partial}{\partial z}\vec{e}_z$

Divergence of a vector $\vec{a}$: $\frac{1}{r}\frac{\partial(ra_r)}{\partial r} + \frac{1}{r}\frac{\partial a_\theta}{\partial \theta} + \frac{\partial a_z}{\partial z} = 0$

Curl of a vector $\vec{a}$: $\nabla \times \vec{a} = \left(\frac{1}{r}\frac{\partial a_z}{\partial \theta} - \frac{\partial a_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial a_r}{\partial z} - \frac{\partial a_z}{\partial r}\right)\vec{e}_\theta + \left(\frac{1}{r}\frac{\partial(ra_\theta)}{\partial r} - \frac{1}{r}\frac{\partial a_r}{\partial \theta}\right)\vec{e}_z$

3. **(15%)** Consider a two-dimensional potential flow field generated by helding a point sink at a distance h above an infinite solid plane. The sink strength is Q.

3% (1) Determine the dimension of Q.

5% (2) Find the pressure distribution along the plane.

7% (3) Compute the total force the sink generates on the plane. Is it a suction or pushing force?

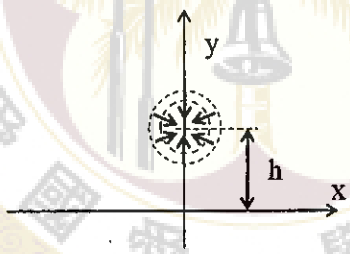

Fig. 3

4. **(25%)** Assuming that you are in charge of an experiment in a space station. (a) Consider an infinitely long, solid, vertical cylinder of radius R in an infinite mass of an incompressible fluid. You would like to analyze first the velocity distribution for the steady flow case in which the cylinder is rotating about a fixed axis with a constant angular velocity ω. (8%) Assume that the flow is axisymmetric and the fluid is at rest at infinity. You may use the Navier-Stokes equations in cylindrical coordinates that can be written as:

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$$\rho\left[\frac{\partial u_r}{\partial t}+(\mathbf{V}\cdot\nabla)u_r-\frac{u_\theta^2}{r}\right]=-\frac{\partial p}{\partial r}+\rho g_r+\mu\left(\nabla^2 u_r-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right)$$

$$\rho\left[\frac{\partial u_\theta}{\partial t}+(\mathbf{V}\cdot\nabla)u_\theta+\frac{u_r u_\theta}{r}\right]=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta+\mu\left(\nabla^2 u_\theta-\frac{u_\theta}{r^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right)$$

$$\rho\left[\frac{\partial u_z}{\partial t}+(\mathbf{V}\cdot\nabla)u_z\right]=-\frac{\partial p}{\partial z}+\rho g_z+\mu\nabla^2 u_z$$

$$\mathbf{V}\cdot\nabla=u_r\frac{\partial}{\partial r}+\frac{u_\theta}{r}\frac{\partial}{\partial \theta}+u_z\frac{\partial}{\partial z}$$

$$\nabla^2=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial z^2}$$

(b) Then a viscous fluid is contained between two infinitely long, vertical, concentric cylinders. The outer cylinder has a radius r_0 and rotates with an angular velocity ω. The inner cylinder is fixed and has a radius r_i. Derive the exact solution for the velocity distribution in the gap (indicating the assumptions). (8 %)

(c) For flow between the concentric cylinders of (b), it is commonly assumed that the tangential velocity (u_θ) distribution in the gap between the cylinders is linear. First obtain a reasonable linear form. (2%) Obviously this is not the same as the exact solution just derived, which is not linear. What are the appropriate dimensionless forms of the tangential velocity and radial position? (2%) For an outer cylinder with r_0 = 5 cm and inner cylinder r_0 = 4.5 cm, show how the dimensionless velocity distribution varies with the dimensionless radial position and compare the difference for the exact and approximate solutions. (5 %)

5. **(25%)** For a flat plate with width w as shown in Fig. 5.1, the drag can be written in terms of the momentum thickness, θ: $D(x)=\rho w U^2\theta = w\int_0^x \tau_w dx$. Given a constant U, one obtains the momentum integral relation for flat-plate boundary-layer flow: $\tau_w=\rho U^2\frac{d\theta}{dx}$, which is valid for either laminar or turbulent flow. (a) It was assumed by von Kármán (1921) that the velocity profiles for laminar flow can be approximated by a parabolic shape: $u(x, y) \approx U(2y/\delta - y^2/\delta^2)$, $0 \le y \le \delta(x)$. Prove that the boundary-layer thickness δ varies with x in the form: $\frac{\delta}{x}\approx\frac{5.5}{\mathrm{Re}_x^{1/2}}$, and therefore obtain a skin-friction coefficient in terms of Re_x along the plate: $c_f=\frac{2\tau_w}{\rho U^2}\approx\frac{0.73}{\mathrm{Re}_x^{1/2}}$, as well as the drag coefficient $C_D\equiv\frac{2D}{\rho U^2 wL}$. (12%)

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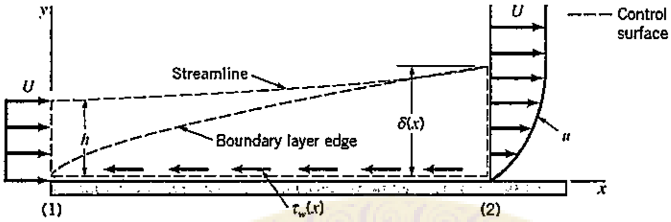

Fig. 5.1

(b) Now consider a square, flat plate shown in Fig. 5.2(a). It is cut into four equal-sized pieces as shown in Fig. 5.2(b). Determine the ratio of the drag on the original plate to that on the plates of the latter case. Assume laminar boundary flow. Interpret the answer physically. (5%)

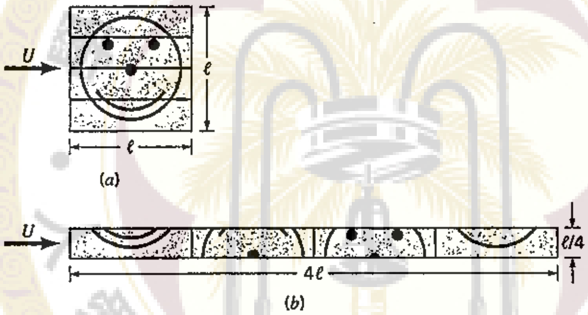

Fig. 5.2

(c) The drag on an airplane cruising at 360 km/h in standard air is to be determined from tests on a 1:10 scale model placed in a pressurized wind tunnel. To minimize compressibility effects, the air speed in the wind tunnel is also to be 360 km/h. Determine the air pressure in the tunnel and the drag on the prototype corresponding to a measured force of 6 N on the model. (8%)

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