國立臺灣大學 102 學年度碩士班招生考試試題

1. (10%) Find the general solutions for y(x) in the following ODE's.
   (a) $y' - a y + b y^2 = 0$
   (b) $y''' - y'' + 4y' - 4y = 0$

2. (a) (5%) If the Laplace transform of $y(t)$ is $Y(s) = \frac{1}{s}\tanh\left(\frac{as}{2}\right)$, where a is a constant, please find the value of y at $t = 0$.
   (b) (5%) Legendre Polynomials are said to be orthogonal on the interval of [-1, 1] because

   $$\int_{-1}^{1} P_m(x)P_n(x)dx = 0 \text{ \&, if } m \neq n \text{, and } \int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}$$

   Supposed that $P_0(x) = 1$ and $P_1(x) = x$, please determine the values of a, b, and c for $P_2(x) = a x^2 + b x + c$ with $a > 0$.

3. (10%) A function is defined as

   $$F(x) = \sum_{n=0}^{\infty} \frac{(-1)^m}{4^m} x^{2m}$$

   (a) Please find the radius of convergence within which the function F(x) has a bounded value. (b) Evaluate the value of F(0.1) to a precision of six digits after decimal place.

4. (10%) A position vector of a particle for time $t > 0$ is $F(t) = \cos(t)\,\mathbf{i} + \sin(t)\,\mathbf{j} + t\,\mathbf{k}$. Please determine (a) the unit tangent vector of the curve F as a function of time, and (b) the radius of curvature of F.

5. (10%) Please use Laplace transform to solve the following differential equation system.

   $$x_1'' - (k x_1 + a x_2') = f, \quad x_1(t=0)=1, \quad x_1'(t=0) = 0,$$
   $$x_2'' + (k x_1 + a x_2') = -f, \quad x_2(t=0)=0, \quad x_2'(t=0) = 1,$$

   where the function f is

   $$f(t) = 1, \quad 0 \le t < 5$$
   $$= 10, \quad 5 \le t.$$

6. (10%) Produce a matrice, **P**, that diagonalizes the given matrice,

   $$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$ that is, $P^{-1}AP$ = diagonal matrix

7. (10%) Find the complex Fourier series expansion of the given periodic function f(t)= 2x, for $0 \le x < 3$ and period 3.

8. (15%) Find the first 5 nonzero terms of the series solution,
   $x^2 y'' - y' + xy = e^x$; $y(-2)=1$ and $y'(-2)=6$

9. (15%) Solve the non-homogeneous heat equation,

   $$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + r \qquad r = \text{constant} \qquad (0 < x < 1,\ t > 0)$$

   $u(0, t) = 0$, $u(1, t) = u_0$ (constant)

   $u(x, 0) = f(x)$

試題隨卷繳回