Problem 1 (15 points) Consider the following two models. The only difference between these two models is that the decision variables, x, in model B are integral.

Model A	Model B
min cx	min cx
s.t. $Ax \le b$	*s.t.* $Ax \le b$
$x \ge 0$	$x \ge 0$ and integer

(a) (4 points) For both models, name one general method of solving each model.

(b) (3 points) Assume that Model A is infeasible? What can you conclude?

(c) (3 points) Assume that Model B is infeasible? What can you conclude?

(d) (5 points) Assume that both models are feasible. Let z^*_A and z^*_B be the optimal objective values for Model A and Model B, respectively. In addition, let z_A be the objective value of a feasible solution to Model A and z_B be the objective value of a feasible solution to Model B. Both z_A and z_B are not optimal. What are the possible relationships among the four values?

Problem 2 (15 points) There are six identical tasks requiring two operations and each operation has one machine for processing arrival tasks. Each task can be processed in any order by the two machines but one task can only be processed on one machine at a time. Similarly, each machine can process one task at a time. All tasks require only one unit of processing time for each operation. In addition, these tasks have arrival times and due dates summarized in the following table. We want to know that "Is it possible to find a schedule in which no jobs are delay?". This question can be answered by solving the problem as a network flow problem. If the network flow problem is feasible, a schedule with no delay jobs exists. Please ***draw*** the corresponding network flow diagram of this problem and clearly indicate all necessary information on the diagram. (You don't need to solve the problem.)

Task	1	2	3	4	5	6
Arrival Time	0	0	1	2	2	3
Due Date	2	3	3	4	5	5

Problem 3 (20 points) A company produces two types of gasoline (gas): regular and premium by distilling from oil. Distilling 1 liter of oil can generate 0.5 liter of regular gas and 0.3 liter of premium gas. The distillation of 1 liter of oil requires 1 machine hour and the cost of purchasing 1 liter of oil is \$2. Regular gas can be sold for \$4 per liter and premium gas can be sold for \$6 per liter. In addition, the company has an option of further processing regular gas into super gas. The further process requires additional 3 machine hours and \$2 to process 1 liter of regular gas into 1 liter of super gas. The price per liter of super gas is \$10. Each week the company has 4000 machine hours available and can purchase up to 3,000 liter of oil. The objective is to maximize the weekly profit. A linear programming model and the corresponding sensitivity report are shown as follows. Answer the following questions and justify your answers.

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Linear Programming Model

[Profit] Max $4x_R + 6x_P + 8x_S - 2x_O$

s.t.

[Oil usage] $x_O \le 3000$

[Machine hours] $x_O + 3x_S \le 4000$

[Regular balance] $0.5x_O = x_R + x_S$

[Premium balance] $0.3x_O = x_P$

$x_R, x_P, x_S, x_O \ge 0$

where x_R, x_P, and x_S denote how many liters of regular, premium, and super gas are produced, and x_O denotes how many liters of oil are used.

Sensitivity Report

Objective (Profit) 6733.333

Variable Cells

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
Oil (x_O)	3000	0	-2	1E+30	0.467
Regular (x_R)	1166.667	0	4	4	0.56
Premium (x_P)	900	0	6	1E+30	1.556
Super (x_S)	333.333	0	8	1.4	4

Constraints

Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Oil Usage	3000	0.467	3000	1000	1400
Machine hours	4000	1.333	4000	3500	1000
Regular balance	0	-4	0	1166.667	1E+30
Premium balance	0	-6	0	900	1E+30

(a) (5 points) If the machine hours can be increased to 4500 hours and only 2500 liters of oil can be purchased, what is the new optimal profit?

(b) (5 points) The company has two options to require **more** resources. Option 1 is to buy at least 1500 liters of oil at $500 from another supplier. Option 2 is to buy a new machine which will provide additional 3500 machine hours at a weekly cost of $4500. Which option is better for the company?

(c) (5 points) The company is offered a new type of oil at $2.5 per liter. Distilling 1 liter of this new oil requires 2 machine hours and will yield 0.6 liter of regular gas and 0.4 liter of premium gas. Should the company purchase this new type of oil?

(d) (5 points) A new technology can distill 1 liter of premium gas into 0.8 liter of super-premium gas and the process requires 3 machine hours. The price of super-premium gas is $13 per liter. Is the company better to adopt this new technology?

Problem 4 (20 points) *NTUIE* has a printer. When the printer breaks down on any given day, *NTUIE* replaces the broken printer with a new one at the beginning of the next day. (i.e. If a printer breaks down anytime on day *t*, a new printer will be put into use at the beginning of day *t*+1.)

Let the life of printers be independent uniform random variables that are uniformly distributed over the interval (3,6). (i.e. The printer is equally likely to fail anytime between 3 to 6 days.) (Uniform random variables are continuous random variables.)

Let X_t be the age of the printer at the beginning of day *t*. (X_t is the time interval between *t* and the last replacement of printer. X_t = 0 if a new printer is put into use at the beginning of day *t*.)

(a) (5 points) Show that X_t is a Markov chain.

(b) (5 points) Given X_t= 4 (the age of the printer is 4 at the beginning of day *t*), find the probability that the printer breaks down on day *t*.

(c) (5 points) Define one step transition probability matrix for this discrete time Markov chain.

(d) (5 points) Given X_8 = 3, find the probability mass function for X_{10}.

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Problem 5 (15 points) Now, *NTUIE* has two printers. One of the printers is *Type 1*, and the other printer is *Type 2*. *NTUIE* waits until both printers are broken and replaces both broken printers immediately with a new *Type 1* printer and a new *Type 2* printer.

The life of printers is independent random variables. The life of each *Type 1* printer is exponentially distributed with mean 5, and the life of *Type 2* printer is exponentially distributed with mean 8.

(a) (10 points) Assume that two new printers are put into use at time 0. Please find the expected time until the first printer failure.

(b) (5 points) Assume that two new printers are put into use at time 0. Please find the expected time until the first replacement of both printers.

Problem 6 (15 points) In Figure 1, the length of each arc is labeled next to the arc.

(a) (15 points) Find the shortest path from node 1 to node 9.

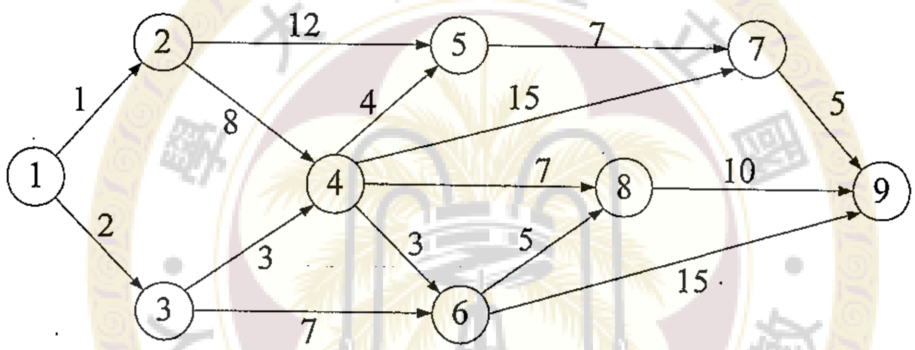

Figure 1

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