1. (10 pts) Find the equation of the tangent line drawn to the graph of $-3x^2 - 16xy - 2y^2 + 3y = 178$ at the point $(-3, 5)$.

2. (10 pts) For what real values of x does the following series converge?

$$\sum_{n=0}^{\infty} \frac{e^n(x-\pi)^{2n}}{\sqrt{n^2+1}}$$

Justify your claim.

3. (15 pts) Compute, with justification, the value of the following limit in terms of α.

$$\lim_{n\to\infty} n^{\alpha} \int_0^{\infty} \exp(-nx^2)dx.$$

(Hint: It might be useful to use the fact that $\int_0^{\infty} t^b \exp(-at)dt = \Gamma(b+1)/a^{b+1}$.)

4. (15 pts) Determine the set of real numbers x for which

$$\sum_{n=1}^{\infty} \left(\frac{1}{n}\csc\frac{1}{n} - 1\right)^x$$

converges.

5. (10 pts) Find, with explanation, the maximum value of $f(x) = x^3 - 3x$ on the set of all real numbers x satisfying $x^4 + 36 \le 13x^2$.

6. (10 pts) Evaluate the integral

$$\int\int_{\Omega} (2x - y)dA$$

where Ω is bounded by the line $y = x$ and the parabola $x = 2 - y^2$.

7. (10 pts) Let R be the parallelogram with vertices $(0,0)$, $(1,1)$, $(2,-1)$ and $(3,0)$. Evaluate the integral $\int\int_R (x+2y)^2 \exp(x-y)dA$.

8. (10 pts) Captain Kurt of the starship Interprize has steered his starship toward a wormhole in space. The Interprize's current location is $(0,2,2)$. The wormhole is quite oddly shaped, and its surface is given by the equation $-x^2+2y^2+2z^2 = 1$. Captain Kurt is really eager to get to the wormhole as soon as possible. Find a point on the wormhole's surface that is nearest to the Interprize so that Kurt knows where to send the ship. Be sure to write up your solution to this problem carefully, explaining all the steps involved in getting to your answer.

9. (10 pts) Let $\mathbf{S}$ be the portion of the unit sphere centered at the origin that is cut out by the cone $z \ge \sqrt{x^2+y^2}$. Evaluate $\int\int_{\mathbf{S}} \mathbf{F} \cdot d\mathbf{S}$ where

$$\mathbf{F}(x,y,z) = (xy + \cos z, -yx + x^2 + z^3, 2z^2 + x).$$

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