

考試科目	數理統計學	所別	統計系	考試時間	2月23日(六) 第 3 節
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- If X is a random variable, $h(x)$ is a non-negative real-valued function, and c is any positive constant. Prove that
 - (10pts) $P(u(X) \geq c) \leq \frac{E(u(X))}{c}$
 - (5 Pts) Use the result in (a) to prove Chebychev inequality

$$P[|X - \mu| \geq k\sigma] \leq \frac{1}{k^2}$$
, for any $k > 0$, where $E(X) = \mu$ and $V(X) = \sigma^2$.
- (10pts) Show that when the population size N approaches infinity, and M/N is kept fixed when N approaches infinity, a Hypergeometric distribution $\text{Hyp}(N, M, n)$ can be approximated by a binomial distribution.
- (10pts) Let X_1, X_2 be independent standard normal random variables, derive the distribution of X_1/X_2 .
- (10pts) Prove that: If X_n converge in probability to X and Y_n converge in probability to Y , then $X_n + Y_n$ converge in probability to $X + Y$.
- Let X_1, \dots, X_n be a random sample from a population with pdf $f(x; \theta) = \theta^{-1} \exp(-x/\theta)$, for $x, \theta > 0$
 - (5pts) Find MLE of θ .
 - (5pts) Find MLE of θ^2 and the asymptotic distribution of this estimator.
 - (10pts) Find the UMVUE of θ .
 - (10pts) Find the MP size α test for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (> \theta_0)$.
 (5pts) Is this test UMP for testing $H_0: \theta = \theta_0$ against $H_1: \theta > \theta_0$? Why or Why not?
 - (10pts) Find the size α LR(Likelihood Ratio) test for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$.
- (10pts) Let X_1, \dots, X_n be a random sample from a Bernoulli distribution, $f(x | \theta) = \theta^x (1-\theta)^{1-x}, x=0,1$, and let $\theta \sim \text{UNIF}(0, 1)$. Find the Bayes estimator with respect to a weighted squared error loss; $L(t; \theta) = (t - \theta)^2 / \theta(1 - \theta)$.

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註 試題隨卷繳交