

考試科目	統計學	所別	經濟學系	考試時間	2月24日(日) 第三節
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1. A random sample is drawn from the density (20分)

$$f(x; \theta) = \theta \left(\frac{x}{2}\right)^\theta.$$

Five observations are 1, 1.5, 0.6, 0.8 and 3. Find the maximum likelihood estimate for θ .

2. $\{X_1, \dots, X_{100}\}$ are i.i.d. random variables with $N(\mu_0, 4)$. (30分)

(a) If the null hypothesis is $\mu_0 = 0$, derive the test statistic and its null distribution.

(b) Please derive the distribution of the test statistic in (a) if $\mu_0 = -2$. In addition, with significance level 5%, compute the type II error and the power of the test. *Note: $\Phi(1.96) = 0.975$.*

3. For a linear probability model (20分)

$$y_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_k x_{ik} + e_i,$$

where y_i is an indicator variable and i.i.d. with mean $E(y_i) = p_i$ and variance $var(y_i) = p_i(1 - p_i)$. Since the model suffers from heteroskedasticity, we can apply a generalized least square procedure. How do you obtain the generalized least square estimates?

4. Consider the following model (30分)

$$y_t = c_0 + y_{t-1} + e_t,$$

with e_t being i.i.d. with mean zero and variance σ_e^2 . Let $y_0 = 0$. Compute the expectation, variance, autocovariance and autocorrelation of y_t .