考試科目	近代物理	所別	應用物理所	考試時間	2月24日(日) 第 二 節

- Planck's constant: $h \approx 6.63 \times 10^{-34}$ joule-sec
- Boltzmann's constant: $k_B \approx 1.38 \times 10^{-23}$ joule/K
- Electron rest mass: $m_e \approx 9.11 \times 10^{-31}$ Kg
- 1 joule $\approx 6.24 \times 10^{18}$ eV

1. [*42 points*] **Short answer questions**

[4] (a) Define *proper length* and *proper time*.

[5] (b) Highlight the important differences between the relativistic Doppler effect and the classical (non-relativistic) Doppler effect.

[5] (c) Write down the time-dependent Schrödinger equation for a free particle of mass m moving in one dimension.

[7] (d) Use the de Broglie relation to find the wavelength of electrons with kinetic energy 10 eV.

[6] (e) How do the energy levels E_n depend on the principal quantum number n for (i) hydrogen atom, (ii) a particle confined in a one-dimensional harmonic potential? (answer without calculation).

[5] (f) For a spinless particle with orbital quantum number $\ell = 2$, what are the allowed magnetic quantum numbers m_ℓ?

[5] (g) Six electrons (spin $s = 1/2$) are confined in the same one-dimensional potential. We ignore the Coulomb repulsion among the electrons. The lowest few single-electron energy levels are shown in Figure 1, where $E_1 < E_2 < \cdots < E_7$. Make a sketch showing how the energy levels would be occupied in the ground state of the six-electron system.

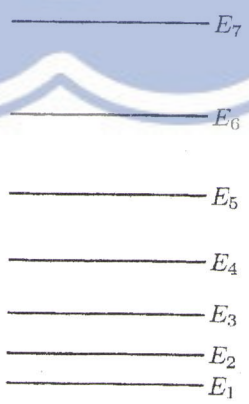

Figure 1

[5] (h) Give the number of protons, neutrons and electrons in a neutral atom of ^{14}C.

備註	試題隨卷繳交

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2. [*14 points*] **Relativity**

A frame S' moves at constant speed v along the positive x direction with respect to the inertial S frame. A thin rod of length L', at rest in the S' frame, makes an angle of θ' with the x' axis, as in Figure 2.

[8] (a) Determine the length of the rod as measured by a stationary observer in the S frame.

[6] (b) Determine the angle θ the rod makes with the x axis.

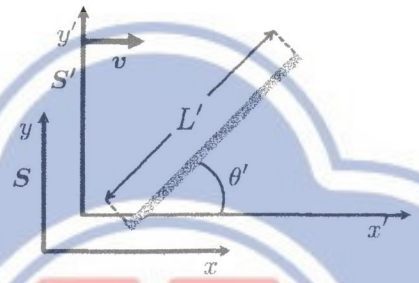

Figure 2

3. [*24 points*] **Schrödinger's theory of quantum mechanics**

Consider a particle confined in a one-dimensional rigid box (*i.e.*, an infinite potential well) of length a. The particle moves freely inside $0 \le x \le a$ but cannot escape outside. Suppose that at time $t = 0$ the particle is described by the wave function

$$\Psi(x, t = 0) = \frac{1}{\sqrt{2}}[\varphi_1(x) + \varphi_2(x)],$$

where $\varphi_1(x)$ and $\varphi_2(x)$ are the two lowest stationary-state wave functions (energy eigenfunctions) for the rigid box, corresponding to the energies E_1 and E_2.

[10] (a) Assuming the stationary-state wave functions $\varphi_1(x)$ and $\varphi_2(x)$ are normalized, verify that the wave function Ψ is also normalized.

[14] (b) Show that the probability density $|\Psi(x, t)|^2$ varies with time, and the time dependence is periodic.

4. [*20 points*] **Quantum particle on a ring**

We consider a particle with mass m that is confined to move on a circle of radius a. Write down the time-independent Schrödinger equation for this particle and solve it to find the eigenstate energies (allowed energies).

備註	試題隨卷繳交