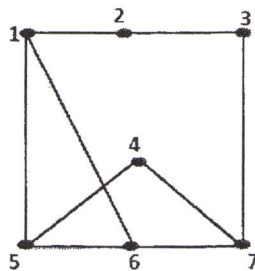
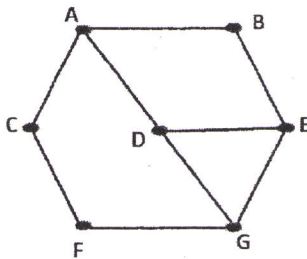


考 試 科 目	計算機數學	系 別	資訊科學系	考 試 時 間	2 月 24 日(日) 第 三 節
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I 離散數學(60%)

- (14%) (a). (4%) Suppose that  $a$  and  $b$  are integers,  $a \equiv 11 \pmod{19}$ , and  $b \equiv 3 \pmod{19}$ . Find the largest integer  $c$  with  $0 \leq c \leq 50$  such that  $c \equiv 2a^2 + 3b^2 \pmod{19}$ .  
 (b). (5%) Find the least positive integer of the multiplicative inverse of 101 modulo 4620.  
 (c). (5%) Find the least positive integer  $x$  satisfying the congruence:  $209^{74} \pmod{53}$ .
- (10%) (a). (4%) Let  $R$  be the relation represented by the matrix  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ . Find the transitive closure of  $R$ .  
 (b). (6%) Find the reflexive closure and the symmetric closure of  $R$  if  $R = \{(1,2), (1,4), (2,3), (3,1), (4,2)\}$ .
- (a). (6%) Find the solution to the recurrence relation  $a_n = 9a_{n-1} - 26a_{n-2} + 24a_{n-3}$  with the initial conditions  $a_0 = 5, a_1 = 19, a_2 = 79$ .  
 (b). (8%) Let  $f(x) = 1/(1-x)^2$ . Find the coefficients  $a_0, a_1, a_2, \dots$  in the expansion  $f(x) = \sum_{k=0}^{\infty} a_k x^k$ .
- (6%) A cubic graph is a simple graph and has every vertex of degree 3.  
 (a). (3%) Draw a cubic graph with 11 vertices, or else prove that there are none.  
 (b). (3%) Draw a cubic graph with 10 edges, or else prove that there are none.
- (8%) Prove or disprove that these two graphs are isomorphic.



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6. (8%) A hash function  $H$  with  $H(k)=h$  is a many-to-one function which maps the input  $k$  to the output  $h$ . In addition,  $k$  is from a large set of variable length, called keys, and  $h$  is from a smaller set of a fixed length. A collision of a hash function means that two different keys are mapping to the same output (ie.,  $H(k_i)=h=H(k_j)$  with  $k_i \neq k_j$ ). A good hash function yields few collisions. Assume there are  $n$  keys and  $m$  is the number of possible outputs of a hash function. What is the probability that there is at least one collision occurred in the hash function? In other words, at least two keys are mapped to the same output. Use  $n$  and  $m$  to express this probability.

II. 線性代數(40%) 請書寫必要的計算過程

(10%) 7. Find the value of  $k$  that satisfies the following equation.

$$\det \begin{bmatrix} 2a_1 & 2a_2 & 2a_3 \\ 3b_1 + 5c_1 & 3b_2 + 5c_2 & 3b_3 + 5c_3 \\ 7c_1 & 7c_2 & 7c_3 \end{bmatrix} = k \det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

(10%) 8. Find all eigenvalues and corresponding eigenvectors of following matrix.

(a)  $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$  (b)  $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

(10%) 9. Find the LU factorization of  $A$ , and solve  $x$  in  $Ax=b$

$$A = \begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} -6 \\ 12 \\ 6 \end{bmatrix}$$

(10%) 10. Let  $V = M_{2 \times 2}(F)$ ,  $W_1 = \left\{ \begin{pmatrix} a & b \\ c & a \end{pmatrix} \in V : a, b, c \in F \right\}$ ,  $W_2 = \left\{ \begin{pmatrix} 0 & a \\ -a & b \end{pmatrix} \in V : a, b \in F \right\}$ .

Find the dimensions of  $W_1$ ,  $W_2$ ,  $W_1 + W_2$ , and  $W_1 \cap W_2$