系所組別： 地球科學系甲、乙組

考試科目： 應用數學

1. Evaluate the following integrals:

   a) $\int \ln x \, dx$　　　　　　(5%)

   b) $\int_0^1 \tan^{-1} x \, dx$　　(10%)

   c) $\int \sec^3 x \, dx$　　　　(10%)

   d) $\int e^x \sin x \, dx$　　　(10%)

2. Find the Taylor series of function $f(x) = \ln x$ at $x = 1$. (10%)

3. Solve the following ODEs:

   a) $(2xy^2 - y)dx + (2x - x^2 y)dy = 0$　　(10%)

   b) $y'' + y' - 2y = 0$　　　　　　　　(5%)

4. Find the general solutions (including homogeneous and nonhomogeneous solutions) of the following ODE which describes a forced oscillation of a spring.

$$my'' + cy' + ky = F_0 \cos\omega t$$

where m is mass of the spring, c is damping constant, k is the spring constant and F_0 is the constant force.　　　　　(15%)

5. The force of attraction,　　　(10%)

$$p = -\frac{c}{r^3}r = -c\left[\frac{x - x_0}{r^3}, \frac{y - y_0}{r^3}, \frac{z - z_0}{r^3}\right]$$

$$r^2 = (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2$$

has the potential

$$f(x, y, z) = \frac{c}{r}$$

i.e.

$$p = \nabla f = \nabla\left(\frac{c}{r}\right)$$

Prove that this potential f is the solution of Laplace's equation

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$$

6. Find the Fourier coefficient of the periodic function $f(x)$ (15%)

$f(x) = -k$ if $-\pi < x < 0$

$f(x) = k$ if $0 < x < \pi$

and $f(x+2\pi) = f(x)$