

系所組別： 資訊工程學系

考試科目： 程式設計

考試日期：0226，節次：2

一、Data Structures (50%)

1. (20%)

Let S_n be the expected number of comparisons in a successful search of a randomly constructed n -node binary tree, and let U_n be the expected number of comparisons in an unsuccessful search. We assume that H_n is the n -th harmonic number. Please represent S_n and U_n respectively using harmonic number.

2. (30%)

For the AOE (Activity on Edge) network described by the table, (a) what is the earliest time the project can finish? (15%) (b) Please list all critical paths. Note that state 1 is the starting state and state 10 is the goal state. (15%)

Activity	From state	To state	Time
a ₁	1	2	5
a ₂	1	3	5
a ₃	2	4	3
a ₄	3	4	6
a ₅	3	5	3
a ₆	4	6	4
a ₇	4	7	4
a ₈	4	5	3
a ₉	5	7	1
a ₁₀	5	8	4
a ₁₁	6	10	4
a ₁₂	7	9	5
a ₁₃	8	9	2
a ₁₄	9	10	2

(背面仍有題目,請繼續作答)

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二、Algorithms (50%)

3. (20%) Solving the recurrence $T(n) = 2T(n/4) + \sqrt{n}$ using Θ notation.

4. (20%) The incident matrix of a directed graph $G = (V, E)$ with no self-loops is a

$|V| \times |E|$ matrix $B = (b_{ij})$ such that

$$b_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves vertex } i, \\ 1 & \text{if edge } j \text{ enters vertex } i, \\ 0 & \text{otherwise.} \end{cases}$$

Describe what the entries of the matrix product BB^T represent, where B^T is the transpose of B .

5. (10%) The Fibonacci numbers are defined by recurrence

$$F_0 = 0,$$

$$F_1 = 1,$$

$$F_i = F_{i-1} + F_{i-2} \text{ for } i \geq 2.$$

Give an $O(n)$ -time dynamic-programming algorithm to compute the n th Fibonacci number.