1. (20%) (a) Find a particular solution to the equation $\dot{x} + 3x = e^{2t}$.

   (b) Find a particular solution to the equation $\dot{x} + 3x = \cos(2t)$.

2. (15%) Consider the 1D wave equation for the waves on the rope:

$$u_{tt} = u_{xx}, \quad 0 < x < 1, \; t > 0 \quad \text{(i)}$$

subjected to the following conditions:

$$u(0,t) = 0 \quad \text{(ii)} \quad \text{and} \quad u(1,t) = \sin \omega t, \; \text{for } t > 0 \quad \text{(iii)}$$
$$u(x,0) = u_t(x,0) = 0, \; \text{for } 0 < x < 1 \quad \text{(iv)}$$

   (a) Find a solution of the wave form

$$U(x,t) = X(x) \sin \omega t$$

   that satisfies the PDE (i) and the BCs (ii), (iii), and (iv).

   (b) Also find where is the rope stationary (i.e. $U(x,t) = 0$)? And (c) for what values of ω is your solution invalid?

3. (15%) Evaluate

$$\int_\gamma \frac{|z|e^z}{z^2} dz$$

where γ is the circle with radius 2 and center 0.

4. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)

   (a) Suppose A and B are square matrices and $AB = O$, where O is the zero matrix. Then either $A = O$ or $B = O$.

   (b) Suppose V is a vector space, and W and U are two subspaces of V. Then the intersection $W \cap U$ is also a subspace of V.

   (c) If all eigenvalues of a matrix A are zero, then $\text{rank}(A) = 0$.

   (d) It is possible that we can define two or more inner products in a vector space.

5. (15%) Let A and B be two square matrices of size n. Which of the following statements are true in general? (Need not to give reasons.) (a) $\text{rank}(AB) = \text{rank}(BA)$.
(b) $AB = BA$. (c) $\det(AB) = \det(BA)$. (d) $\text{tr}(AB) = \text{tr}(BA)$.
($\det(M)$ and $\text{tr}(M)$ denote the determinant and the trace of a square matrix M, respectively.)

6. (15%) Denoted by $\mathcal{M}_n$ the vector space of all $n \times n$ matrices, where n is an integer. Suppose that S is a subset of $\mathcal{M}_n$ and S is composed of all non-invertible matrices. Determine if S is a subspace of $\mathcal{M}_n$. (You need to verify or prove your answer.)