1. (15%) A bar of length L is broken into three pieces at two random spots. What is the probability that the length of at least one piece is less than $L/20$?

2. (10%) Let $p(x,y,z) = (xyz)/162$, $x = 4,5$, $y = 1,2,3$, and $z = 1,2$, be the joint probability mass function of the random variables X, Y, Z.
   (a) Calculate the joint marginal probability mass functions of X, Y.
   (b) Find $E(YZ)$.

3. (15%) Let X be a continuous random variable with set of possible values $\{x : 0 < x < \alpha\}$ (where $\alpha < \infty$), distribution function F, and density function f. Using integration by parts, prove the following expectation

$$E[X] = \int_0^\alpha [1 - F(t)]dt$$

relationship.

4. (10%) In a study conducted three years ago, 82% of the people in a randomly selected sample were found to have good financial credit ratings, while the remaining 18% were found to have bad financial credit ratings. Current records of the people from that sample show that 30% of those with bad credit ratings have since improved their ratings to good, while 15% of those with good credit ratings have since changed to having a bad credit rating. What percentage of people with good credit ratings now had bad ratings three years ago?

5. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)
   (a) If all eigenvalues of a matrix A are zero, then $\text{rank}(A) = 0$.
   (b) Suppose A and B are square matrices and $AB = O$, where O is the zero matrix. Then either $A = O$ or $B = O$.
   (c) Suppose V is a vector space, and W and U are two subspaces of V. Then the intersection $W \cap U$ is also a subspace of V.
   (d) It is possible that we can define two or more inner products in a vector space.

6. (15%) Let A and B be two square matrices of size n. Which of the following statements are true in general? (Need not to give reasons.) (a) $\det(AB) = \det(BA)$. (b) $AB = BA$. (c) $\text{tr}(AB) = \text{tr}(BA)$. (d) $\text{rank}(AB) = \text{rank}(BA)$. ($\det(M)$ and $\text{tr}(M)$ denote the determinant and the trace of a square matrix M, respectively.)

7. (15%) Denoted by $\mathcal{M}_n$ the vector space of all $n \times n$ matrices, where n is an integer. Suppose that S is a subset of $\mathcal{M}_n$ and S is composed of all non-invertible matrices. Determine if S is a subspace of $\mathcal{M}_n$. (You need to verify or prove your answer.)