一、單選題(共 30 分)：每題有五個選項，其中只有一個最適當的答案，每題答
　　對得 5 分；未作答、答錯或答多於一個選項者，該題以 0 分計算。

1. Consider the discrete-time complex exponential $x[n] = e^{j2\pi n/N}$, N is a positive integer. Which of the following statements about $x[n]$ is false ?

   (a) Its fundamental period is N.

   (b) It is orthogonal to $e^{j2\pi \cdot 2n/N}$.

   (c) It is orthogonal to $e^{j2\pi \cdot 3n/N}$.

   (d) Its discrete-time Fourier transform is $X(j\omega) = 2\pi\,\delta(\omega - \frac{2\pi}{N})$.

   (e) Its discrete-time Fourier transform is a periodic function.

2. Suppose that continuous-time $x(t)$ is a periodic signal with period T, and, its associated Fourier series coefficients are $a_k = \frac{1}{T}\int_0^T x(t)\,e^{j2\pi kt/T}\,dt, \forall k$. Which of the following statements about $x(t)$ is false ?

   (a) It can be written as $\sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$.

   (b) Its spectrum can be expressed as $\sum_{k=-\infty}^{\infty} 2\pi\,a_k\,\delta\left(\omega - \frac{2\pi}{T}k\right)$.

   (c) Its autocorrelation is defined by $R_x(\tau) = \lim_{x\to\infty} \int_{t=-\infty}^{\infty} x(t)x^*(t-\tau)\,dt$.

   (d) Its autocorrelation is equal to $R_x(\tau) = \sum_{k=-\infty}^{\infty} |a_k|^2 e^{j2\pi k\tau/T}$.

   (e) Its time-averaged power is equal to $\sum_{k=-\infty}^{\infty} |a_k|^2$.

3. Suppose that $x(t) = x_c(t)\cos(2\pi f_0 t) - x_s(t)\sin(2\pi f_0 t)$ is a real-valued bandpass signal at center (or carrier) frequency f_0. Which of the following statements about $x(t)$ is false ?

   (a) Its lowpass equivalent signal is $x_c(t) + j\,x_s(t)$.

   (b) Its corresponding analytic signal $x(t) + j\,\hat{x}(t)$, where $\hat{x}(t)$ denotes the Hilbert transform of $x(t)$.

(c) Its lowpass equivalent signal can be obtained by

$$x_\ell(t) = LP\{2x(t)\cos(2\pi f_0 t)\} - j\,LP\{2x(t)\sin(2\pi f_0 t)\},\ \text{where}\ LP\{\cdot\}$$

denotes the lowpass filtering.

(d) It is related to its analytic signal $z(t)$ by $x(t) = \dfrac{1}{2}\big[z(t) + z^*(t)\big]$.

(e) Its Hilbert transform produces $\hat{x}(t) = x_c(t)\sin(2\pi f_0 t) - x_s(t)\cos(2\pi f_0 t)$.

4. An *i.i.d.* discrete time random process X_n has mean m and variance σ^2.

   Which of the following statements is true ?
   (a) The random variable X_n is Gaussian distributed.
   (b) The mean of random variable $Y = \displaystyle\sum_{n=1}^{N} X_n$ is m.
   (c) The variance of random variable $Y = \dfrac{1}{N}\displaystyle\sum_{n=1}^{N} X_n$ is σ^2
   (d) The process X_n is wide-sense stationary.
   (e) The process $Z_n = X_n - X_{n-1}$ is also an *i.i.d.* discrete time random process.

5. Which of the following statements about bandwidth efficiency is true ?
   (a) The bandwidth efficiency of the double sideband suppressed carrier (DSB-SC) AM is higher than that of conventional AM
   (b) The bandwidth efficiency of the conventional AM is higher than that of FM
   (c) The bandwidth efficiency of the single sideband AM is the same as that of the conventional AM.
   (d) The bandwidth efficiency of binary PSK is higher than that of binary ASK
   (e) The bandwidth efficiency of binary PSK is the same as that of quaternary PSK

6. Let $N(t)$ be a zero-mean white Gaussian noise with power spectral density $N_0/2$. Which of the following statements is true ?

   (a) $\displaystyle\int_0^1 N(t)\,dt = 0$.

   (b) The power of $N(t)$ is finite.

   (c) The power spectral density of $N(t)$ is $\dfrac{N_0}{2}\delta(t)$ for some N_0.

   (d) If $N(t)$ is passed through an LTI system, the output of the LTI system is

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also a white Gaussian random process.

(e) If $N(t)$ is sampled at t_1 and t_2, then $N(t_1)$ and $N(t_2)$ are independent Gaussian random variables

二、計算題(共 40 分)：

1. (10 分) Consider the three waveforms $\psi_n(t)$ shown in Fig. 1.

   (a) Show that the waveforms are orthogonal.

   (b) Express the waveform $y(t)$ as a weighted linear combination of $\psi_1(t)$, $\psi_2(t)$, and $\psi_3(t)$, if

$$y(t) = \begin{cases} -1, & 0 \le t \le 1 \\ 1, & 1 \le t \le 3 \\ -1, & 3 \le t \le 4 \end{cases}$$

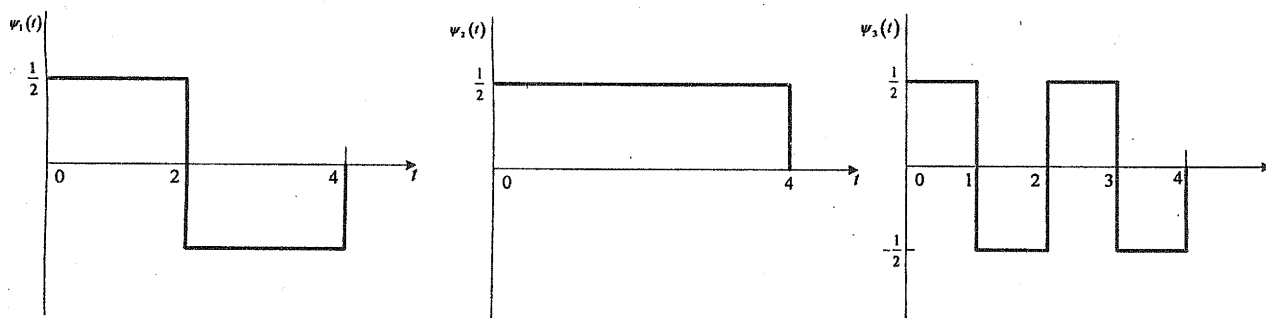   and determine the weighting coefficients.

Figure 1: Three waveforms

2. (10 分) Consider the signal detector with an input

$$r = \pm A + n$$

   where $+A$ and $-A$ occur with equal probability and the noise n is random with the Laplacian probability density function

$$p(n) = \frac{1}{\sqrt{2}\sigma} e^{-|n|\sqrt{2}/\sigma}.$$

   Determine the probability of error as a function of parameters A and σ.

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3. (10 分) The message signal $m(t) = 10\,\mathrm{sinc}(500t)$ frequency modulates the carrier $c(t) = 100\cos(2\pi f_c t)$. The modulation index is 5.
   (a) Write an expression for the modulated signal.
   (b) What is the maximum frequency deviation of the modulated signal ?
   (c) What is the power content of the modulated signal ?
   (d) Find the bandwidth of the modulated signal ?

4. (10 分) The random process $X(t)$ is defined by $X(t) = X\cos(2\pi f_0 t)$ where X is a zero-mean standard Gaussian random variable.
   (a) Find the time and ensemble average of $X(t)$
   (b) Find the autocorrelation function $R_X(t+\tau, t)$ of $X(t)$. Is $X(t)$ stationary ? Is it cyclostationary ?
   (c) Find the power-spectral density of $X(t)$.

三、名詞解釋(共 30 分)：請利用數學符號、數學式、圖表或其他專業術語寫兩段短文(每段至多 500 字)，分別解釋下列兩個名詞。

1. (15 分) Optimal Detection of BPSK Signals
2. (15 分) Sampling Theory