

系所組別： 國際企業研究所

考試科目： 微積分

考試日期： 0225 · 節次： 3

1. Find the following derivatives. (10%)

(a) $\frac{d}{dx} \ln |x^3 + 2^x|$, at $x=1$ (b) $D_x e^{\sin(x^2+x)}$

2. Find the following integrals. (20%)

(a) $\int \frac{t}{\sqrt{1-t^4}} dt$ (b) $\int x^2 (\ln x)^2 dx$

(c) $\int_0^\infty \frac{x+1}{e^{3x}} dx$ (d) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

3. Test $\int_1^4 \frac{1}{(x-2)^2} dx$ for convergence. (10%)

4. Find the area between the curves $y = 12 - 3x^2$ and $y = 4x + 5$ from $x=0$ to $x=3$. (10%)

5. Verify that $\int_1^{ab} \frac{1}{t} dt = \int_1^a \frac{1}{t} dt + \int_1^b \frac{1}{t} dt$, $\forall a, b > 0$ (10%)

6. Beginning 1 month from now, each month \$ 250 will be deposited into an account where the interest is compounded continuously at the annual rate of 9 percent. Use a definite integral to approximate the amount of money in the account immediately after the 36th deposit. (10%)

7. The present value of the continuous stream of income $C(t)$ dollars per year, where t is the number of years from now, for T years at continuous interest rate r is $\int_0^T C(t)e^{-rt} dt$. A business generates income at the rate of $2t$ million dollars per year, where t is the number of years from now. Find the present value of this continuous stream for the next five years at the continuous interest rate of 10%. (15%)

8. A college textbook publishing firm finds that its yearly profit is

$$P(x, y) = -0.5x^2 - 3y^2 - 2xy + 206x + 612y$$

hundreds if each year it sends x thousand complimentary copies of its book professors and employs y sales representatives. The high point on the graph of $P(x, y)$ is the highest point. How many complimentary copies and how many sales representatives should it employ to maximize the yearly profit? (15%)