You must show all your works in order to get all credit.

1. X has distribution function $F(x)=\begin{cases} ce^{x} & \text{if } x<0 \\ p+c(2-e^{-x}) & \text{if } x\geq 0 \end{cases}$, **where** $0<p<1$.

(8%) (a) Find the value of c and $f(x)$, **the pdf of X.**

(8%) (b) Find the expectation E(X) and the variance Var(X).

(8%) (c) Find the moment generating function of X.

2. Let X and Y have joint probability density function (pdf)

$f(x,y)=1;\ 0<x<1,\ 0<y<1$, **and zero otherwise.**

(7%) (a) Find the joint pdf of U= X+Y and V=X-Y.

(7%) (b) Find the marginal pdf of U.

3. Let $X_1,\ldots,X_n$ **be iid Poisson(**λ**), and let** λ **have a gamma(**α,β**) distribution with E(**λ**)=**$\alpha\beta$.

(7%) (a) For squared error loss, find the Bayes estimator of λ.

(7%) (b) Find the Bayes risk of the Bayes estimator.

4. Let $X_1,\cdots,X_n$ **be a random sample of size n from a distribution with pdf**

$f(x;\eta,\theta)=\theta^{-1}e^{-(x-\eta)/\theta},\ x>\eta$.

(8%) (a)Find the maximum likelihood estimate (MLE) $\hat{\theta}$ **of** θ **and MLE** $\hat{\eta}$ **of** η.

(8%) (b)Show that $\hat{\theta}$ **and** $\hat{\eta}$ **are independent.**

(8%) (c)Find the uniformly minimum variance unbiased estimate of η.

(8%) (d)Find a 1-α confidence interval of θ **by pivotal methods.**

(8%) (e)Find the distribution of (n-1)($X_{(1)}$**-**η**)/**$\hat{\theta}$**, where** $X_{(1)}$**=min(**$X_1,\cdots,X_n$**).**

(8%) (f)Find the critical region for a size α generalized likelihood ratio test of

$H_0:\eta\leq\eta_0$ **versus** $H_a:\eta>\eta_0$.