1. Find the following derivatives. (10%)

(a) $\frac{d}{dx}e^{|x^3+3^x|}$ (b) $D_x \sin(\ln x^3)$

2. Find the following integrals. (10%)

(a) $\int \sin^2 3x \cos^4 3x \, dx$ (b) $\int_0^\infty \frac{x+\sqrt{x}}{e^{2x}} \, dx$

3. Find the following limits. (10%)

(a) $\lim_{x\to 0+}(e^x + 2x)^{3/x}$ (b) $\lim_{n\to\infty} \sum_{k=0}^{n} \frac{2^k}{k!}$

4. Test the following series for convergence.

(a) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\tan^{-1} k}$ (b) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} \ln k}{k}$ (10%)

5. Find the area between the curves $y = 12 - 3x^2$ and $y = 4x + 5$ from $x = 0$ to $x = 3$. (10%)

6. Use double integral to calculate the area Ω enclosed by $y = x^2$ and $x + y = 2$. (10%)

7. Let T be a solid with volume

$V = \int\int_T\int dxdydz = \int_0^2 \int_0^{9-x^2} \int_0^{2-x} dzdydx.$

Find the values of a, b, c in the following representations.

(a) $V = \int_0^a \int_0^b \int_0^c dydxdz.$

(b) $V = \int_0^5 \int_0^2 \int_0^a dzdxdy + \int_5^9 \int_0^b \int_0^c dzdxdy.$ (10%)

(背面仍有題目,請繼續作答)

8. Determine whether the quadratic form
$Q = 2(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3 - x_1x_3)$
is positive definite or semi-positive definite. (10%)

9. Find the set containing the largest possible number of independent vectors among the following vectors. (10%)

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad \text{and} \quad v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$$

10. Suppose you know that the 3 by 4 matrix A has the vector $s = (2, 3, 1, 0)$ as the only special solution to $Ax = 0$.
(a) What is the rank of A? (2%)
(b) What is the exact row reduced echelon form R of A? (2%)
(c) What is the column space of A? (2%)
(d) How do you know that $Ax = b$ can be solved for all b? (2%)
(e) What is the left nullspace of A? (2%)