

國立交通大學 101 學年度碩士班考試入學試題

科目：工程數學(1801)

考試日期：101 年 2 月 16 日 第 1 節

系所班別：台南分部光電學院聯招

組別：台南光電聯招

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【不可使用計算機】*作答前請先核對試題、答案卷(試卷)與准考證之所組別與考科是否相符!!

1. Solve $\frac{dy}{dx} = \frac{[y + xy^3(1 + \ln x)]}{x}$ (15%)

2. Solve $y'' - 2y' + 2y = 2e^x(\cos x + x)$ (10%)

3. Solve $(3x+2)y'' + 3(3x+2)y' - 36y = 3x^2 + 4x + 1$ (10%)

4. For the matrix $\begin{bmatrix} 1 & 0 & 0 \\ -8 & 4 & -6 \\ 8 & 1 & 9 \end{bmatrix}$, find its eigen values and eigen vectors (10%)

5. Find the least-squares fit to data set below by a linear function $y = f(x) = ax + b$.

Data set: $(x_1, y_1) = (0, 1)$, $(x_2, y_2) = (2, 6)$, $(x_3, y_3) = (3, 11)$, and $(x_4, y_4) = (4, 12)$.

(Reminder: During least-squares fit, the following condition must be satisfied -

$\sum_{i=1}^n (y - y_i)^2$ has a minimum value.) (10%)

6. Given a matrix of $A = \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 5 & -6 & 0 \\ -1 & -2 & 1 & -1 \\ 4 & 10 & -9 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 8 \\ 17 \\ -8 \\ 33 \end{bmatrix}$, and $Ax = b$

(a) Using LU factorization to find matrices L (lower triangular matrix) and U (upper triangular matrix). (5%)

(b) Solve $Ax = b$ using L and U. (5%)

7. (a) Evaluate $\int_0^{\infty} \frac{\ln(x^2 + 1)}{x^2 + 1} dx$ (10%)

(b) Evaluate $\int_2^{\pi} \ln(\sin x) dx$ (10%)

8. z is a complex variable, integrate $\frac{z^3 + 2z + i}{z^3 - 1}$

(a) in the counterclockwise sense around a circle of radius 1 with center at the point $z = 1$ (5%)

(b) in the counterclockwise sense around a circle of radius 1 with center at the point $z = i$ (10%)