1. (20%) Find the characteristic polynomial and the eigenvalues for the matrix

$$D = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}.$$

2. (20%) Find A^{-1}, $A+B$, $A-B$ and AB where the matrices

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & 1 \\ 4 & 1 & 9 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 5 & 0 & -2 \\ 0 & -4 & 1 \\ 2 & 1 & 6 \end{bmatrix}.$$

3. (10%) (a) Write down the definition of $tr(A)$, the trace of a $n \times n$ square matrix A.

(10%) (b) Find the trace of $I + D + D^2 + \cdots + D^{30}$ where

$$D = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$$

4. (20%) Let $S = \{u_1, u_2, u_3\}$ be the basis for R^3, where $u_1 = (2, 0, 1)$, $u_2 = (0, 3, -2)$, and $u_3 = (-1, 1, 4)$. Use the Gram-Schmidt process to transform S to an orthonormal basis for R^3.

5. Let $F : R^3 \rightarrow R^2$ and $G : R^2 \rightarrow R^3$ be linear transformations defined by

$$F\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -2x_1 + x_2 + 3x_3 \\ 2x_1 - x_2 + x_3 \end{bmatrix}$$

and

$$G\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 3x_2 \\ x_1 + x_2 \\ -x_1 + x_2 \end{bmatrix}.$$

(a)(10%) Give a formula for $G \circ F$.

(b)(10%) Find matrices A, B and C such that $F(x) = Ax$, $G(x) = Bx$, and $[G \circ F](x) = Cx$.