國立中山大學113學年度
碩士班暨碩士在職專班招生考試試題

科目名稱：工程數學【材光系碩士班選考、材料前瞻應材碩士班選考、材光聯合碩士班選考】

—作答注意事項—

考試時間：100分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆(含鉛筆)書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶(未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以2B鉛筆劃記，不可使用修正液（帶）塗改，未使用2B鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- 試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。
- 試題採雙面列印，考生應注意試題頁數確實作答。
- 違規者依本校招生考試試場規則及違規處理辦法處理。

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科目名稱：工程數學【材光系碩士班選考、材料前瞻應材碩士班選考、材光聯合碩士班選考】題號：488001

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　　　共1頁第1頁

Quantitative calculation problems (100 points). You are required to provide a thorough explanation of your calculations and reasoning. Please show all of your work and clearly demonstrate how you arrived at your solutions.

1. Solve the following first-order differential equation $\cot x\mathrm{d}y + y\mathrm{d}x = 0$ with the initial condition: $y(0) = 1$ (10 pt).

2. Show that the general solution to the first-order differential equation: $\frac{\mathrm{d}y}{\mathrm{d}x} + P(x)y = Q(x)$ is $y(x) = [\int I(x)Q(x)\mathrm{d}x + c]/I(x)$, where $I(x) = \exp[\int P(x)\mathrm{d}x]$ and c is a constant (15 pt).

3. Solve the differential equation: $x\frac{\mathrm{d}y}{\mathrm{d}x}+3y = \frac{\cos x}{x^2}$ with the initial condition: $y(0) = 0$ (10 pt).

4. Solve the differential equation: $x^2y^4 + y + 3x\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. with $y(0) = 1$ (10 pt).

5. Solve the differential equation: $2y'' + 6y' - 20y = 120e^{5x}$ with $y(0) = 7$ and $y'(0) = -1$ (10 pt).

6. Solve the differential equation: $y'' - 4y' + 8y = 4e^{2x}\sec 2x$ with $y(0) = 0$ and $y'(0) = 0$ (15 pt).

7. Solve the system of first-order differential equations:

$$\begin{cases} \dot{x} = 2x - 4y \\ \dot{y} = x - 3y \end{cases},$$

where the solution satisfies $x(0) = 9$ and $y(0) = 3$ (15 pt).

8. Assuming both of the Laplace transforms of $f(t)$ and $g(t)$ exist and are denoted as $\mathcal{L}\{f(t)\} = F(s)$ and $\mathcal{L}\{g(t)\} = G(s)$, respectively, prove that:
(A) $\int_0^t f(x)g(t-x)\mathrm{d}x = \int_0^t f(t-x)g(x)\mathrm{d}x$ (5 pt);
(B) $\mathcal{L}\{\int_0^t f(x)g(t-x)\mathrm{d}x\} = F(s)G(s)$ (5 pt),
and (C) calculate $\mathcal{L}\{\int_0^t e^{-ax}\sin[\omega(t-x)]\mathrm{d}x\}$, where a is a negative constant (5 pt).

試題請隨卷繳回，請留意背面是否有題