國立中山大學 113 學年度

碩士班暨碩士在職專班招生考試試題

科目名稱：普通物理【材光系碩士班選考、材料前瞻應材碩士班選考、材光聯合碩士班選考】

—作答注意事項—

考試時間：100 分鐘

- **考試開始鈴響前不得翻閱試題，並不得書寫、劃記、作答。**請先檢查答案卷（卡）之應考證號碼、桌角號碼、應試科目是否正確，如有不同立即請監試人員處理。
- **答案卷限用藍、黑色筆 (含鉛筆) 書寫、繪圖或標示**，可攜帶橡皮擦、無色透明無文字墊板、尺規、修正液（帶）、手錶 (未附計算器者)。每人每節限使用一份答案卷，請衡酌作答。
- 答案卡請以 2B 鉛筆劃記，不可使用修正液（帶）塗改，未使用 2B 鉛筆、劃記太輕或污損致光學閱讀機無法辨識答案者，後果由考生自負。
- 答案卷（卡）應保持清潔完整，不得折疊、破壞或塗改應考證號碼及條碼，亦不得書寫考生姓名、應考證號碼或與答案無關之任何文字或符號。
- 可否使用計算機請依試題資訊內標註為準，如「可以」使用，廠牌、功能不拘，唯不得攜帶書籍、紙張（應考證不得做計算紙書寫）、具有通訊、記憶、傳輸或收發等功能之相關電子產品或其他有礙試場安寧、考試公平之各類器材入場。
- **試題及答案卷（卡）請務必繳回，未繳回者該科成績以零分計算。**
- **試題採雙面列印，考生應注意試題頁數確實作答。**
- **違規者依本校招生考試試場規則及違規處理辦法處理。**

國立中山大學113學年度碩士暨碩士專班招生考試試題

科目名稱：普通物理【材光系碩士班選考、材料前瞻應材碩士班選考、材光聯合碩士班選考】
題號：488005
※ 本科目依簡章規定「不可以」使用計算機（問答申論題）　　　　共 2頁第 1 頁

Problem 1. [Mechanics: 35 points]

There are two atoms A and B, one with the position $\mathbf{x}_A$ and mass m_A, and the other with the position $\mathbf{x}_B$ and mass m_B.

(a) [5 points] There is an electrostatic force $\mathbf{F}_{BA}$ exerted by atom A on atom B , and a force $\mathbf{F}_{AB}$ exerted by atom B on atom A. According to Newton's third law, $\mathbf{F}_{BA} = -\mathbf{F}_{AB}$. If we define the relative displacement $\mathbf{x} = \mathbf{x}_A - \mathbf{x}_B$, the relative motion can be described by the equation of motion:

$$\mathbf{F}_{AB} = \mu \frac{d^2\mathbf{x}}{dt^2},$$

where μ is called "reduced mass". Prove

$$\mu = \frac{m_A\, m_B}{m_A + m_B}$$

(b) [5 points] The potential energy between the two atoms A and B can be approximated as the Lennard-Jones potential

$$U(x) = \epsilon\left[\left(\frac{\sigma}{x}\right)^{12} - 2\left(\frac{\sigma}{x}\right)^{6}\right]$$

where ϵ and σ are constants. Calculate the distance x_0 wherein the potential energy $U(x_0)$ is the minimum of $U(x)$. x_0 is the bond length of the molecule AB.

(c) [10 points] Around the minimum of $U(x)$, the potential energy is approximately a parabolic curve. Do Taylor's expansion of $U(x)$ around $x = x_0$ to the second order.

(d) [5 points] The force can be derived from the potential energy by $F_{AB} = -\frac{dU}{dx}$. Using the Taylor's expansion obtained in the previous problem, calculate the approximate force around the minimum. The force you obtain should look like the Hooke's law $F = -kx$. What is the k? (expressed you answer with those given physical quantities $\mathbf{x}_A, m_A, \mathbf{x}_B, m_B, \epsilon, \sigma$)

(e) [10 points] The two atoms are bound with approximately a Lennard-Jones potential, with a bond length x_0. The Lennard-Jones force is like a spring with a elastic constant k. Therefore, the system is 2 balls coupled with a spring. Using the k you obtain in the previous problem, calculate the vibration (relative motion) angular frequency ω of the molecule AB (expressed you answer with those given physical quantities $\mathbf{x}_A, m_A, \mathbf{x}_B, m_B, \epsilon, \sigma$).

Problem 2. [Electromagnetism: 35 points]

The electric field at a distance $\mathscr{r}$ from a point charge Q is:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\mathscr{r}^2}\hat{\mathscr{r}},$$

where $\hat{\mathscr{r}}$ is the unit vector along the vector $\mathscr{r}$. The electric potential is:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\mathscr{r}}$$

Consider the following questions in the vacuum, and **use the units in the above formula.**

(a) [10 points] Find the electric field inside a solid sphere of radius R that carries a uniform volume charge density ρ. Express your answer in terms of the total charge of the sphere, q.

(b) [5 points] An atom can be approximately thought as a point nucleus ($+q$) surrounded by a uniformly charged solid spherical cloud ($-q$) of radius R. If there is an external electric field $\mathbf{E}_{\text{ext}} = E_{\text{ext}}\,\hat{x}$, the nucleus would move in the $+\hat{x}$ direction while the electron sphere would move in the opposite direction. If the distance between the nucleus and the center of the electron sphere is d, calculate the electric field $\mathbf{E}_{\text{electron}}$ exerted by the electron sphere to the nucleus.

(c) [10 points] Following the previous problem, there are (1) the external field $\mathbf{E}_{\text{ext}}$, and (2) the electric field $\mathbf{E}_{\text{electron}}$ exerted by the electron sphere, onto the neucleus. If the two electric fields cancel each other, $\mathbf{E}_{\text{ext}} = -\mathbf{E}_{\text{electron}}$, then the atom system reaches equilibrium. Calculate the distance d at equilibrium.

(d) [10 points] An electric dipole is composed of a charge $+q$ at $\mathbf{d}/2$, and a charge $-q$ at $-\mathbf{d}/2$. The dipole moment $\mathbf{p}$ is defined as $\mathbf{p} = q\mathbf{d}$. If there is a uniform external electric field $\mathbf{E}_{\text{ext}}$, prove that the torque

$$\mathbf{N} = \mathbf{p} \times \mathbf{E}_{\text{ext}}.$$

Problem 3. [Thermodynamics: 15 points]

A gas consisting of n moles of a monoatomic gas goes through the cycle shown in the Figure 1, wherein the pressure is constant for A → B, the volume is constant for B → C, and the temperature is constant for C → A.

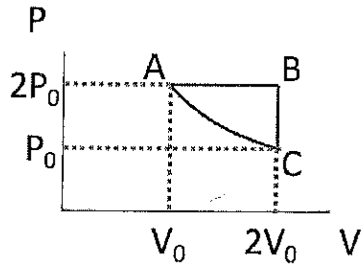

Figure 1

Calculate the heat transfer Q (Q = Δ U + W) for (a) A → B, (b) B → C, and (c) C → A.

Problem 4. [Waves/Optics/Modern physics: 15 points]

If a propagating wave along the x direction has the form $f(x,t) = Ae^{i(kx-\omega t+\phi)}$, where x is the position, t is the time, A is a constant amplitude and ϕ is a constant phase.

(a) [5 points] Calculate $f(x, t_0 + T)$, what is the difference between $f(x, t_0 + T)$ and $f(x, t_0)$?

(b) [5 points] The wave is a periodic function of the time, so $f(x, t_0) = f(x, t_0 + T)$ for arbitrary t_0 (where T is the period). Use this relation, express the period T with A, k, ω, and ϕ.

(c) [5 points] (i) Calculate $[f^*(x) \cdot f(x)]$. (* denotes the complex conjugate) (ii) Calculate $\frac{\partial^2 f}{\partial x^2}$. (iii) Calculate $\int_0^\lambda f(x,t)dx$, where λ is the wave length.