國立成功大學
113學年度碩士班招生考試試題

編　號：174

系　所：電機工程學系

科　目：線性代數

日　期：0201

節　次：第 3 節

備　註：不可使用計算機

編號：174 **國立成功大學 113 學年度碩士班招生考試試題**

系　所：電機工程學系

考試科目：線性代數　　　　考試日期：0201，節次：3

第1頁，共1頁

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (30 pts, 3 pts each) Mark each statement True or False (2 pts for correct answer). Justify each answer (1 pts).
   a. If **U** is $m \times n$ with orthogonal columns, then $\mathbf{U}\mathbf{U}^T\mathbf{x}$ is the orthogonal projection of **x** onto Col **U**.
   b. If **B** is $m \times n$ and **x** is a unit vector in $\Re^n$, then $\|\mathbf{Bx}\| \le \sigma_1$, where σ_1 is the first singular value of **B**.
   c. A singular value decomposition of an $m \times n$ matrix **B** can be written as $\mathbf{B} = \mathbf{P\Sigma Q}$, where **P** is an $m \times m$ orthogonal matrix, **Q** is an $n \times n$ orthogonal matrix, and $\mathbf{\Sigma}$ is an $m \times n$ "diagonal" matrix.
   d. If **W** is a subspace, then $\|\text{proj}_W\, \mathbf{v}\|^2 + \|\mathbf{v} - \text{proj}_W\, \mathbf{v}\|^2 = \|\mathbf{v}\|^2$.
   e. A least-squares solution of $\mathbf{Ax} = \mathbf{b}$ is the vector $\mathbf{A}\hat{\mathbf{x}}$ in Col **A** closest to **b**, so that $\|\mathbf{b} - \mathbf{A}\hat{\mathbf{x}}\| \le \|\mathbf{b} - \mathbf{Ax}\|$ for all **x**.
   f. The normal equations for a least-squares solution of $\mathbf{Ax} = \mathbf{b}$ are given by $\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$.
   g. If **A** is row equivalent to the identity matrix **I**, then **A** is diagonalizable.
   h. Each eigenvector of an invertible matrix **A** is also an eigenvector of $\mathbf{A}^{-1}$.
   i. If **A** is $m \times n$ and the linear transformation $\mathbf{x} \mapsto \mathbf{Ax}$ is onto, then rank $\mathbf{A} = m$.
   j. A change-of-coordinates matrix is always invertible.
2. (16 pts, 8 pts each) Let $T: \Re^n \to \Re^m$ be a linear transformation.
   a. What is the dimension of the range of T if T is a one-to-one mapping? Explain.
   b. What is the dimension of the kernel of T if T maps $\Re^n$ onto $\Re^m$? Explain.
3. (16 pts, 8 pts each)
   a. In the vector space of all real-valued functions, find a basis for the subspace spanned by $\{\sin t, \sin 2t, \sin t \cos t\}$.
   b. The set $B = \{1 - t^2, t - t^2, 2 - 2t - 6t^2\}$ is a basis for P_2. Find the coordinate vector of $\mathbf{p}(t) = 3 + t - 6t^2$ relative to B.
4. (20 pts) Find the Singular Value Decomposition of $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$. That is, write $\mathbf{A} = \mathbf{U\Sigma V}^T$.
5. (18 pts) Find a least-squares solution (8 pts) and its least-squares error (10 pts) of $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A} = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}$.